# The precursors to stick-slip events in sheared granular systems

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## Introduction

The stick-slip transition of granular systems is related to earthquakes and avalanches, and therefore understanding the conditions leading to slip events is of general importance. Although stick-slip behavior has been studied extensively, what triggers a slip event still remains unclear. The purpose of our study is to explore the existence of precursors to slip events.

We study a sheared system in stick-slip regime via two-dimensional discrete element simulations.

Particular focus is on the evolution of force networks before and during slip events. We will show that some features of force network evolution could be used to gain insight into the occurrence of a slip event.

Jiaojia Fault Gauge

Earthquakes in Italy

## Discrete Element Simulations

- Soft spheres/disks interacting via normal and tangential forces
- The method allows for realistic simulations of a number of different systems
- Normal forces: linear springs
- Tangential forces: Cundall-Strack model

### Governing Equations:

$$m_i \frac{d^2\mathbf{r}_i}{dt^2} = m_i\mathbf{g} + \mathbf{F}^n_{i,j}$$

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = -\frac{1}{2} d_i \mathbf{n}_i \times \mathbf{F}^t_{i,j}$$

$$\mathbf{n}_i = \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|}$$

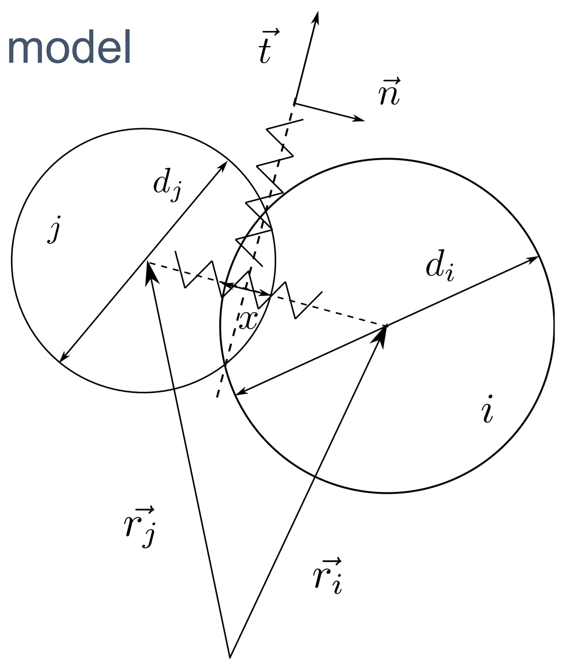
where,

$$\mathbf{F}^n_{i,j} = k_n x_{i,j}\mathbf{n} + \gamma_n \bar{m}\mathbf{v}^n_{i,j}$$

$$\mathbf{F}^{t*}_{i,j} = -k_t\boldsymbol{\xi} + \gamma_t \bar{m}\mathbf{v}^t_{i,j}$$

$$\boldsymbol{\xi} = \boldsymbol{\xi}' - \mathbf{n}(\mathbf{n}\cdot\boldsymbol{\xi}'), \boldsymbol{\xi}' = \int_{t_0}^{t} \mathbf{v}^t_{i,j}(t')dt'$$

$$\mathbf{F}^t_{i,j} = \min(\mu|\mathbf{F}^n_{i,j}|, |\mathbf{F}^{t*}_{i,j}|)\mathbf{F}^{t*}_{i,j}/|\mathbf{F}^{t*}_{i,j}|$$



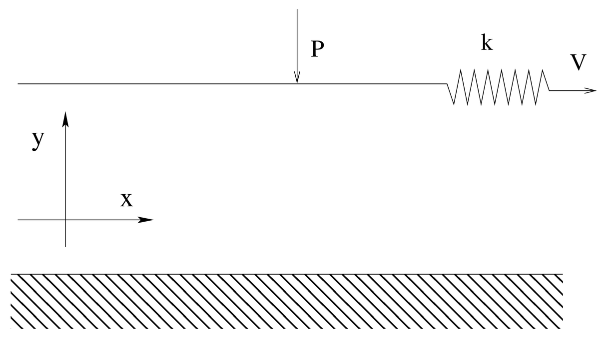

Particle contact

## Persistence Homology

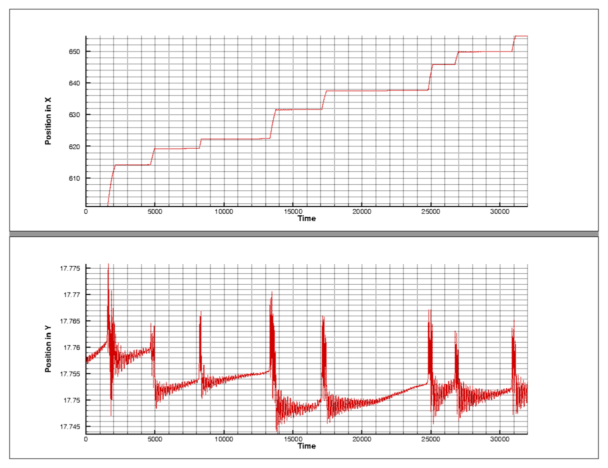
- Use topology based approach to carry out data reduction: from large time dependent data sets to simpler well-defined mathematical structures
- Important point: the resulting mathematical structures still contain the most important physics of the considered systems and therefore their analysis allows to reach new insights into the physical properties of the considered systems
- Basic idea: strength of the interaction between particles is crucial; filtering force networks by varying force thresholding provides
    1. important information about the system
    2. the means for analysis of spatial and temporal properties of the force networks
- Persistence diagrams capture complete information about force landscape

## Simulation & Results

We use MD simulations to carry out the 2D stick-slip simulations

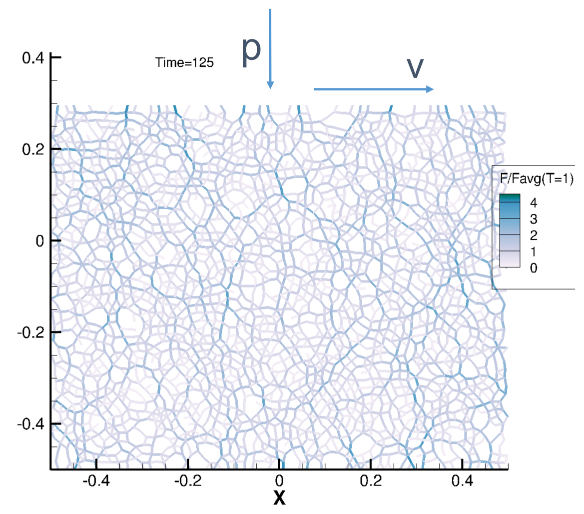



Geometry setting

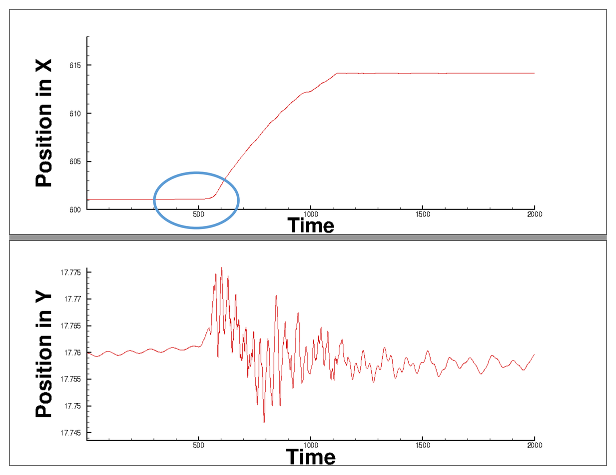


Top wall position

For a single slip event, the force network and top wall position:




Force Network

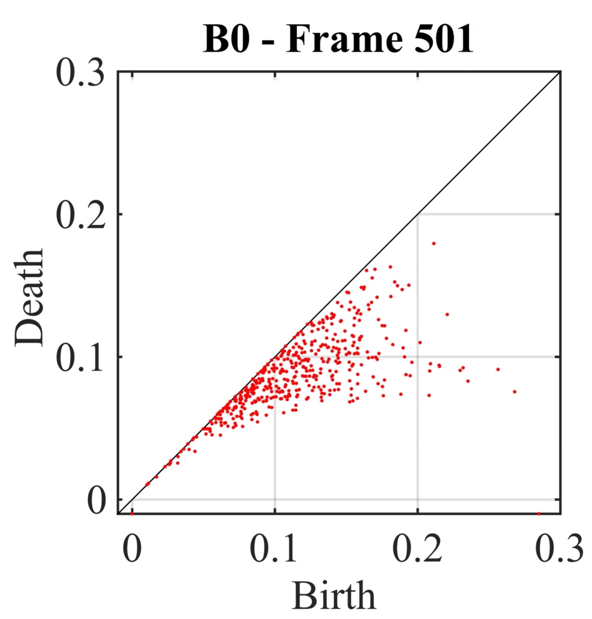



Wall position of a single slip event

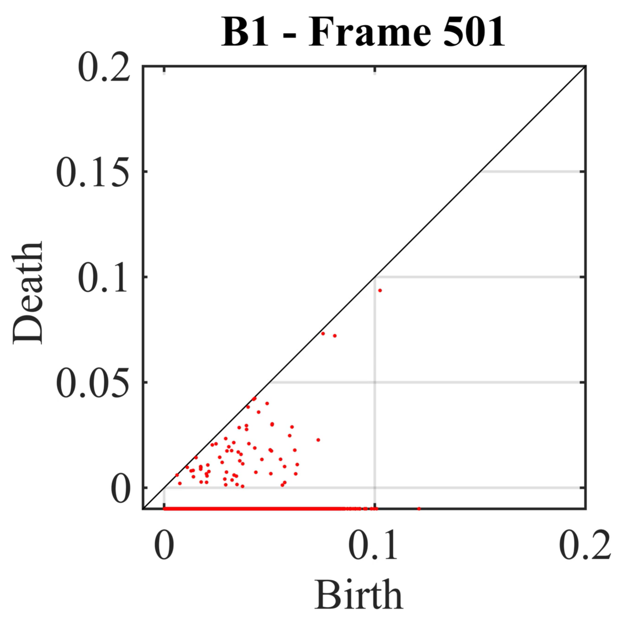
## Persistence Diagrams

Persistence diagrams capture complete information about force landscape. In these diagrams, Betti numbers are used to characterize the geometry and development of force networks:

- Zeroth Betti Number $\beta_0$ - number of connected components
- First Betti Number $\beta_1$ - number of loops/voids




$\beta_0$ persistence diagram




$\beta_1$ persistence diagram

## Persistence Distances

Persistence diagrams live in a metric space and therefore can be compared

Our approach is to compute the distance between all points in a diagram, and match the points so that this distance is minimized. Examples of the norms that can be used:

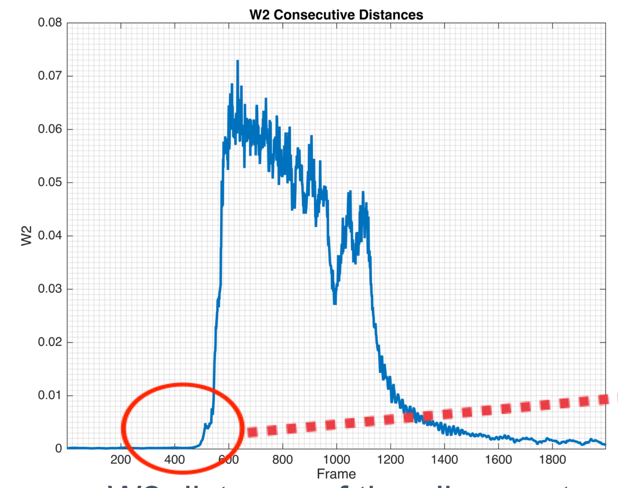
- Largest distance only, as bottleneck distance:

$$d_B(PD_n(f), PD_n(g)) = \sup_{p\in PD_n(f)} \|p - \gamma(p)\|_\infty$$

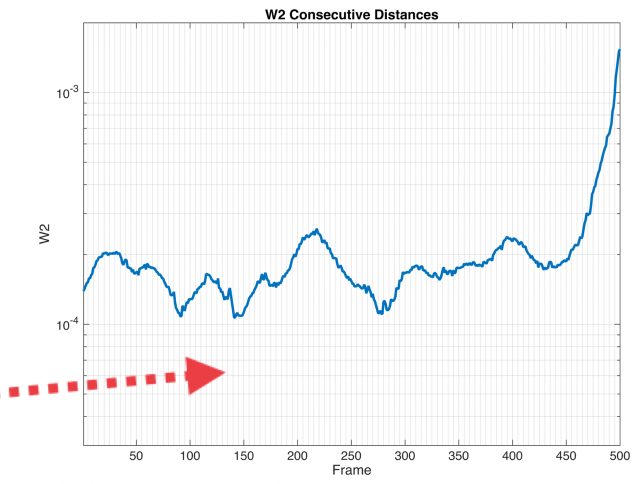
- All the distances and include them with appropriate weights, Wasserstein $q$-distance:

$$d_{W1}(PD_n(f), PD_n(g)) = \left(\sum_{p\in PD_n(f)} \|p - \gamma(p)\|^q_\infty\right)^{1/q}$$

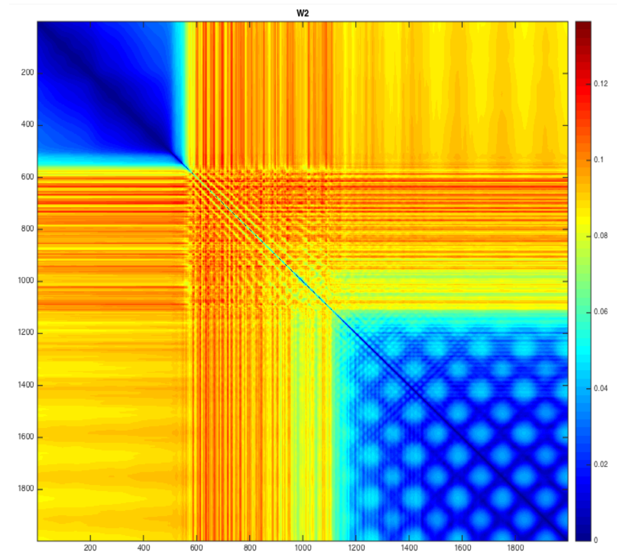
Here, $q = 2$.



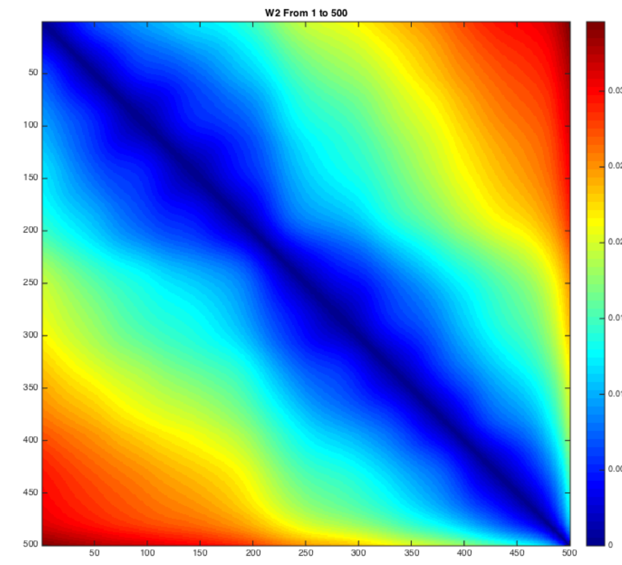

W2 distance of the slip event



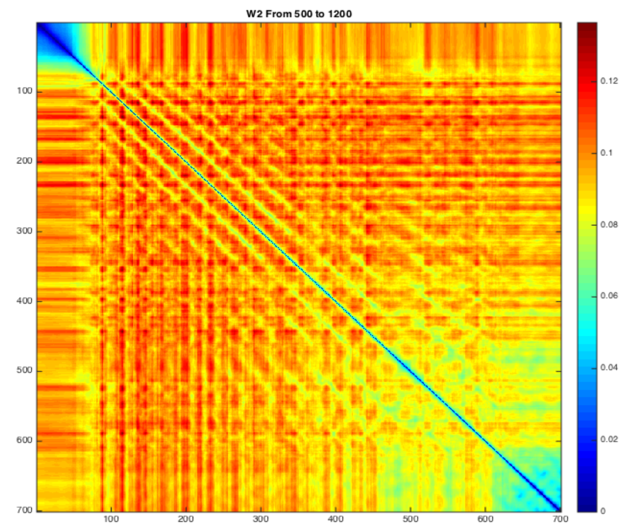

W2 distance for the first 500 frames



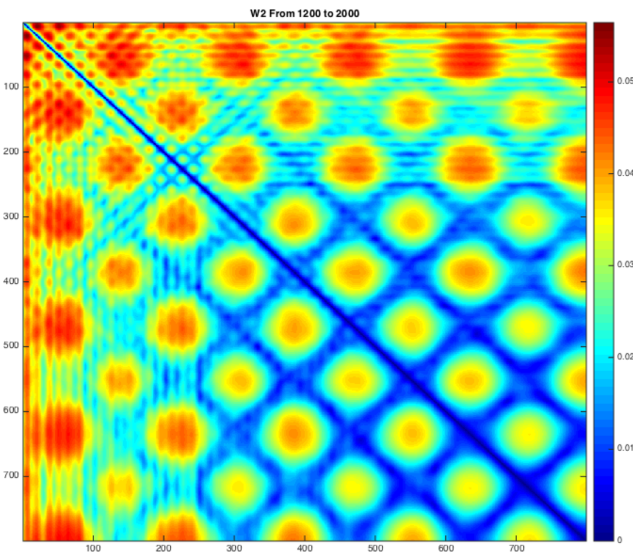
W2 distance matrix of the slip event




W2 distance matrix for the first 500 frames




W2 distance matrix for the frame 500-1200




W2 distance matrix for the frame 1200-2000

## Conclusions

- Topology based methods provide a way to simplify considerably quantitative description of force networks in sheared granular systems
- Based on the simplified description, we are able to quantify the connection between mesoscale information (force networks) and macroscopic system response (slip)
- Force networks analysis suggests existence of precursors to slip events
- What remains to be done:
    1. carry out analysis of a large number of slip events
    2. describe more precisely the precursors and their properties

## Reference

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## Acknowledgment

This work is supported by ARO, NSF Grant No. DMS 1521717, DARPA Contract HR0011-16-2-0033.