Pleated membrane filters, which offer larger surface area to volume ratios than unpleated membrane filters, are used in a wide variety of applications. However, the performance of the pleated filter, as characterized by a flux-throughput plot, indicates that the equivalent unpleated filter provides better performance under the same pressure drop. Earlier work (Saniei & Cummings, 2016) used a highly-simplified membrane model to investigate how the pleated membrane geometry affects this performance difference. In this work we extend this line of investigation, and use asymptotic methods to couple an outer problem for the flow within the pleated structure to an inner problem that accounts for the detailed pore structure within the membrane. By adding this level of detail, we are able to observe a decrease in particle retention by the membrane under certain conditions, which has been observed experimentally. We use our model to formulate and address questions of optimal membrane design for a given filtration application.

2. Mathematical Model

We consider pleated filters with high peak packing density (FPD), so that we may assume that all flow regions are occupied by either porous support material or filter membrane, with no air gaps. Figure 1 indicates how we simplify the geometry, first by neglecting dependence on the coordinate z that runs parallel to the pleat tisps/valleys, and then if considering the cylindrical cartridge) neglecting the curvature of the cylinder. This reduces the problem to 2D in the (X,Y) plane, where X is the direction along the pleat (from tip to valley), and Y is the coordinate perpendicular to the membrane in each pleat. We further simplify by neglecting the curvature at the pleat tip and valley, which allows us to view each pleat as one of a periodic array of rectangles. We assume each rectangle (pleat) is symmetric about the centerline (Y=0), which gives our simplified half-pleat domain, indicated in Figure 2.

Figure 1: Left: sketch of our pleated membrane filter; Right: Cross Section. Schematic showing a few pleats. The region between two dotted lines indicates a single complete pleat, assumed to repeat periodically. The arrows indicate the three-layer structure, with gray grey denoting the support layer and dark grey the membrane layer in reality much thinner than the dotted layers. Full arrow indicates the flow direction. Light arrow is a single period axis, indicating how the geometry is identified in the model with the same color coding as the arrows.

2.1 Modeling Assumptions and Governing Equations

We consider pleated filters with high peak packing density (FPD), so that we may assume that all flow regions are occupied by either porous support material or filter membrane, with no air gaps. Figure 1 indicates how we simplify the geometry, first by neglecting dependence on the coordinate z that runs parallel to the pleat tisps/valleys, and then if considering the cylindrical cartridge) neglecting the curvature of the cylinder. This reduces the problem to 2D in the (X,Y) plane, where X is the direction along the pleat (from tip to valley), and Y is the coordinate perpendicular to the membrane in each pleat. We further simplify by neglecting the curvature at the pleat tip and valley, which allows us to view each pleat as one of a periodic array of rectangles. We assume each rectangle (pleat) is symmetric about the centerline (Y=0), which gives our simplified half-pleat domain, indicated in Figure 2.

Figure 2: Schematic diagram of the pleat, showing boundary conditions at inlet and outer and schematic coordinates. Symmetry is assumed within each pleat Y=0. X denotes the membrane centerline coordinate, with each pore assumed to lie in a square centered on a line Y=0 at X=x.

We assume that Darcy's law governs flow in both the support layers and the filter membrane. Both are porous media, though the support material has a much higher permeability (k) than that of the membrane (k_m). Support layer permeability is assumed to be constant in both space and time, reflecting an assumption that fouling of support layers does not occur. The velocity \( U \) is defined as the average flux through the support layer is then given in terms of the pressure \( P(X,Y) \) by:

\[
U = \nabla P = \frac{\partial P}{\partial X} = \frac{\partial P}{\partial Y} = 0
\]

3. Results

3.1 Definitions for Filter Performance Evaluation and Optimization

To evaluate filter performance and to carry out design optimization, we first define some key quantities:

Netux = \int \left[ \sigma(x,t) \right] \, dt

\[
\text{throughput}(t) = \int \nabla U \, \cdot \, \nabla P \, \sigma(x,t) \, \, \partial X \, \partial Y / \partial P
\]

throughput = \int \left[ \sigma(x,t) \right] \, dt

\[
\text{cumulate} = \int \left[ \sigma(x,t) \right] \, dt
\]

3.2 Effect of \( \lambda \) on Optimization

We optimize for the initial pore profile \( \sigma(x,t) \) within the class of low-order polynomials (linear, quadratic, cubic). Figures 3(a-c) show results for polyurethane on the classes of linear, quadratic, and cubic polynomials. We used a multiobjective optimization method to find the optimal pore profile that minimizes a set of goals.

4. Conclusions

We have formulated a general membrane optimization problem based on maximizing throughput subject to a particle removal constraint.

5. Acknowledgments

The authors acknowledge financial support from the NSF under grant DMS-1615719.

References