Analyzing Force Networks in Granular Systems using Persistent Homology

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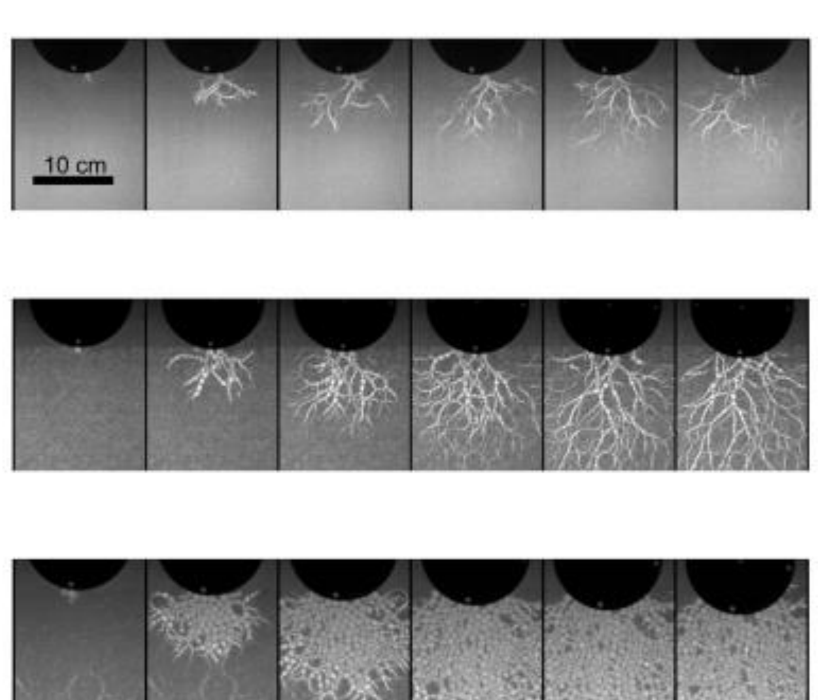
Abstract

We present novel approach that allows to correlate in precise terms the dynamics of an intruder falling on a granular system and the material response. Our results are obtained using **topological data analysis** method, **persistent homology,** for analysis of the results of experiments that involve impact on a system of granular photoelastic particles. The photoelasticity illuminates the forces that develop in the two dimensional **granular system**. These forces form a complex network that evolves as the intruder penetrates the system. Persistent homology allows us to quantify the topological features of the force network based on the experimental images. We cross validate the information obtained from the images to the physical measurements from the experiment. We find, for multiple experimental settings, that the total persistence of topological generators is highly correlated with the acceleration of the intruder thus showing direct connection between the topological properties and the material response.

Introduction

We study the force propagation that occurs in granular systems after experiencing impact. This has applications in missile defense and terrestrial locomotion over granular terrain. The images that we use are from the granular impact experiments conducted at Duke University [1]. In the experiments, intruders of different sizes were dropped onto a system of photoelastic particles of varying stiffness to determine a relation between the physical parameters of granular systems and force propagation speed. Our analysis of the granular system is centered around the topology of the force network.



This figure shows the grayscale images from the impact experiment; from top to bottom the stiffness of the photoelastic particles decreases. The camera captures optical properties of the particles, related to the stress that they experience. From [1].

Method

For each experiment, we are given a set of grayscale images $\{A_k \in \mathbb{N}^{m\times n} \mid 0 \le a_{ij} \le 255\}$, where k is a discrete time index and $m \times n$ is the resolution of the image. Each image is represented by a matrix of integers ranging from 0 to 255, where 0 is black and 255 is white. We start by applying image processing techniques to reduce noise in the images. Then we apply super level persistent homology to the image A to decompose it into a table of topological generators: consider a Boolean matrix B consisting only of 1s and 0s. We call any adjacent cluster of 1s a component and any cluster of 0s surrounded by 1s a hole. They are collectively called generators. The method performs a continuous mapping of A to a simplicial complex B_ρ for the threshold $\rho = 255\ to\ 0$, where

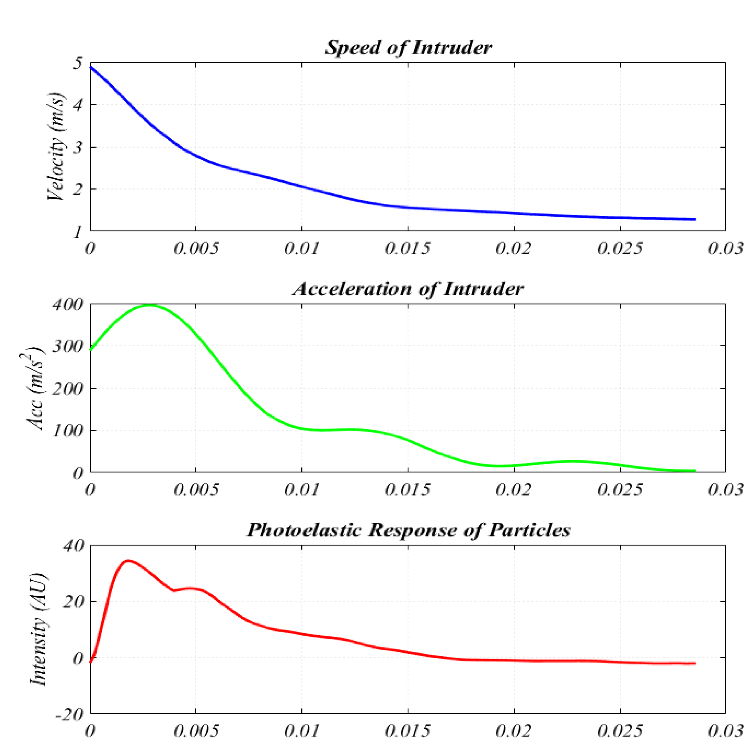
$$(B_\rho)_{ij} = \begin{cases} 1 & a_{ij} > \rho \\ 0 & a_{ij} \le \rho \end{cases}.$$

Persistent homology tabulates the thresholds ρ at which holes and components appear and disappear on the B_ρ matrix as ρ is swept from 255 to 0. For a given image A_k, we have a set of generators described by their birth and death thresholds, $\mathrm{PD_k} = \{pd_i \in \mathbb{Z}^{2\times 1}\}_{i=1}^{n_k}$, and we call this the persistence diagram of the image.

There are several ways to analyze persistence diagrams, see [2-4] for more details:

- *Death vs. Birth plot*
- *Distribution of lifespan* (birth – death)
- *Betti numbers* (count of components and holes)
- *Total persistence* (sum of lifespans)

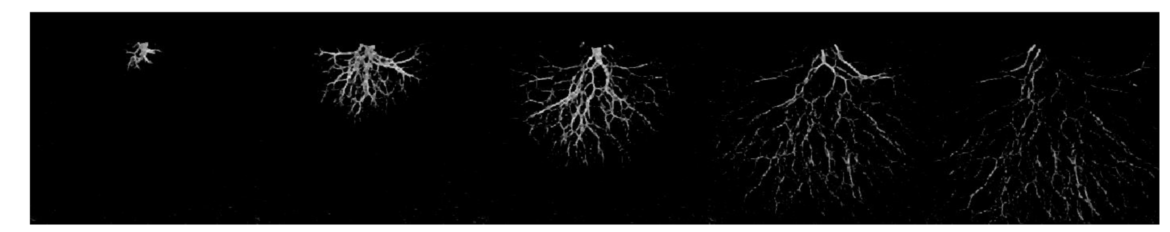
We ask whether there is a connection between these measures and the dynamics of the system. The experimental data for the base case is plotted below.



We consider the case with medium particle stiffness, high impact speed (4.0 m/s), and small intruder size (2.5 in). The velocity and acceleration of the intruder are recorded. The acceleration and brightness are closely related because the amount of light refracted by the particles is proportional to the amount of stress exerted onto them.

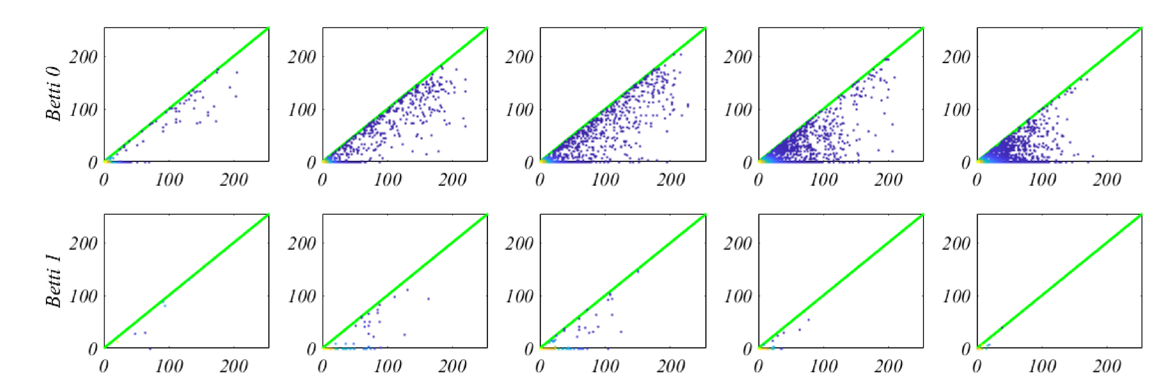
Results

Processed Images



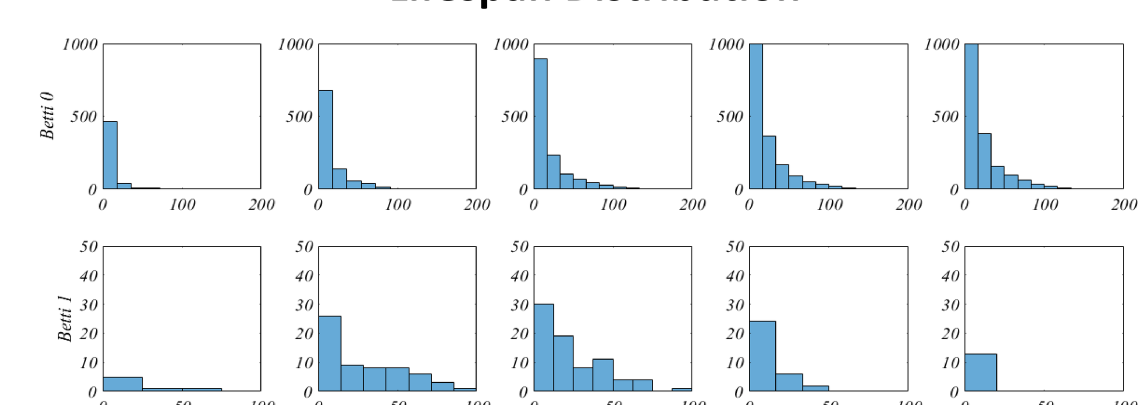
Base experiment: medium stiffness, small intruder size, and high impact speed. Images were processed to isolate the force network. 5 frames were chosen.

Birth vs. Death Plot



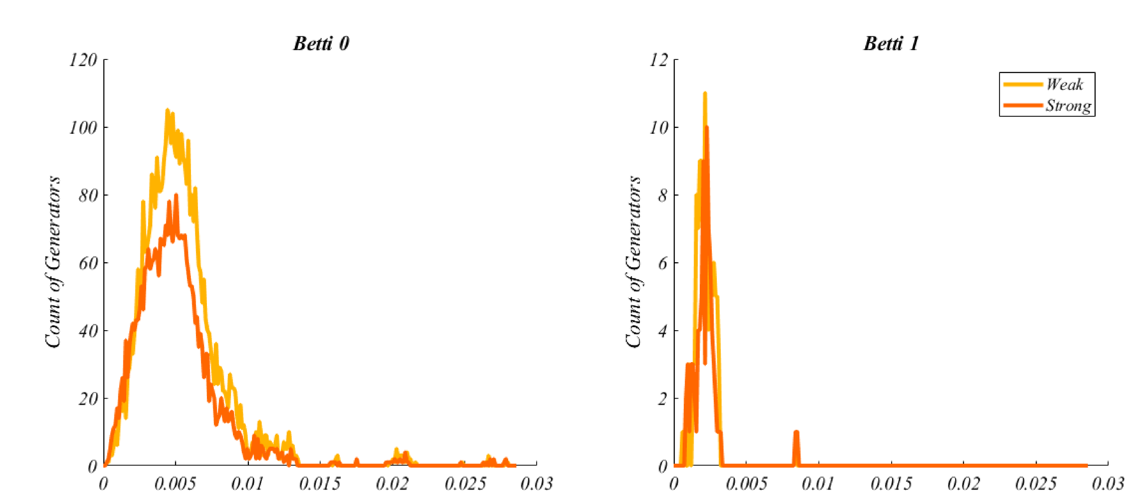
The scatter plot of generators for β_0 and β_1 is plotted for the five frames shown above.

Lifespan Distribution



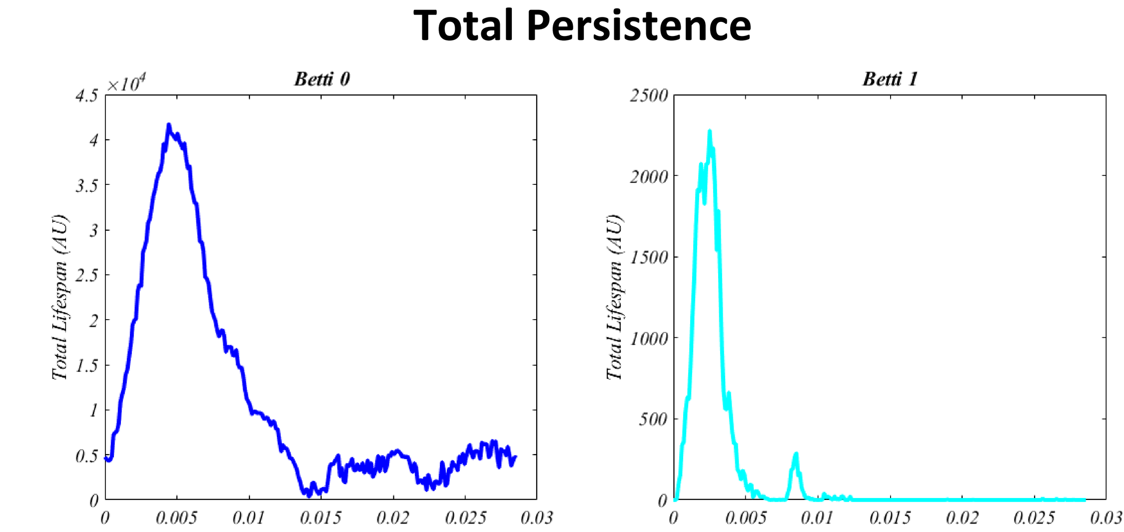
Histogram of the lifespan of generators is plotted for the five frames shown above.

Betti Numbers



The number of generators above and below the average generator birth threshold. The counts correspond to strong and weak forces in the network.

Total Persistence

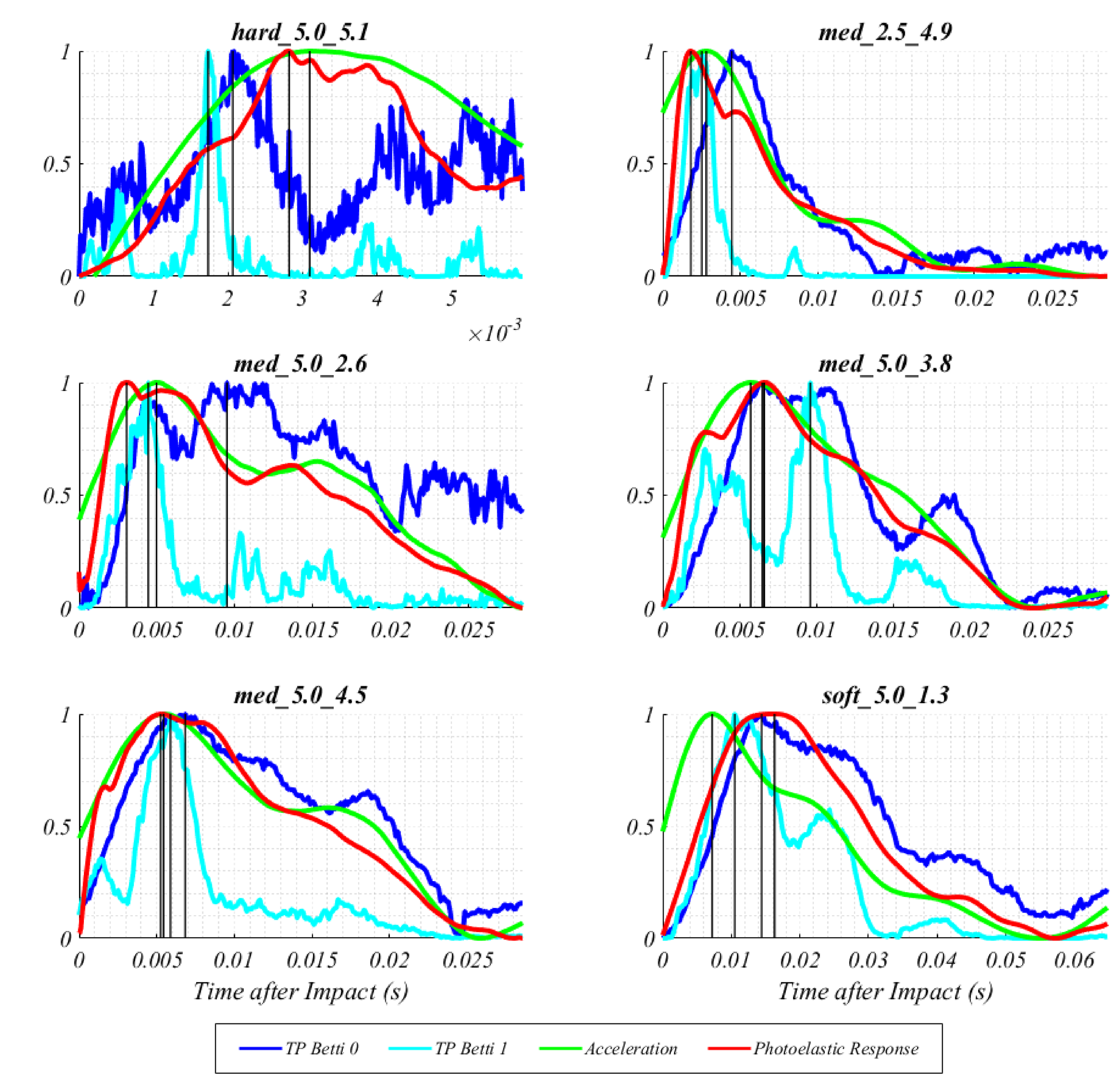


The sum of the generator lifespans.

Conclusion

We have found across several experiments that the acceleration of the intruder and the photoelastic response follow the same trend as the total persistence and Betti numbers.

The results were normalized and plotted in a single window for 6 different impact experiments. We observe good correlation between various measures, particularly for the particles of the smaller stiffness.



The research in the direction of comparing the results to the dynamics is in progress. Another direction currently being studied is the relation between the evolution of persistence diagrams and the speed of information propagation through granular particles.

Acknowledgements

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References

[1] Clark, A.H., Petersen, A.J., Kondic, L., Behringer, R.P.: Nonlinear Force Propagation During Granular Impact. Physical Review Letters. 114, (2015).

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[3] Kramar, M., Goullet, A., Kondic, L., Mischaikow, K.: Persistence of force networks in compressed granular media. Physical Review E. 87, (2013).

[4] Kramár, M., Goullet, A., Kondic, L., Mischaikow, K.: Evolution of force networks in dense particulate media. Physical Review E. 90, (2014).