NJIT Grace Hopper AI
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Homogeneity Analysis of Porous Media via Persistent Homology

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Abstract

3D images of porous materials often exhibit topologically homogeneous structure meaning any sufficiently large sub-volume statistically resembles any other. Our project investigates this homogeneity hypothesis using persistent homology (PH) on a very large 3D binary image of a porous medium. We aim to determine how topological features (like connected components and loops) scale with sample size and whether their normalized measures converge, indicating a representative volume. The motivation is both scientific and computational: confirming homogeneity justifies analyzing smaller subdomains instead of the entire huge dataset, significantly reducing computational cost. A broad audience takeaway is that by using topology, we can quantitatively confirm the material's uniformity, which in turn validates using smaller samples for simulations or analyses.

Methodology

Data[3]**:** The input data is a very large 3D binary image of a porous material (solid vs void). To test homogeneity, we extracted a series of cubic subdomains of increasing side length L (L is the side length of the cube in voxels, so the cube's volume is L^3 voxels). We used sizes 50^3, 100^3, 150^3, … up to 600^3 voxels.

The smaller cubes are nested within larger ones (overlapping), so each larger cube contains the previous smaller region – this simulates observing the material with an expanding window. We also later consider disjoint non-overlapping cubes to check spatial uniformity. These non-overlapping cubes let us compare different physical locations in the sample, not just different sizes. To carry out PH on a large 3D image, we use two complementary approaches:

Alpha Complexes (via GUDHI)[1]**:** We treat the discrete solid and void voxel coordinates as point clouds and compute an alpha complex filtration, which is a simplicial complex built on the points at increasing radius. Using the GUDHI library, we obtain persistence diagrams for features in the solid and void phases.

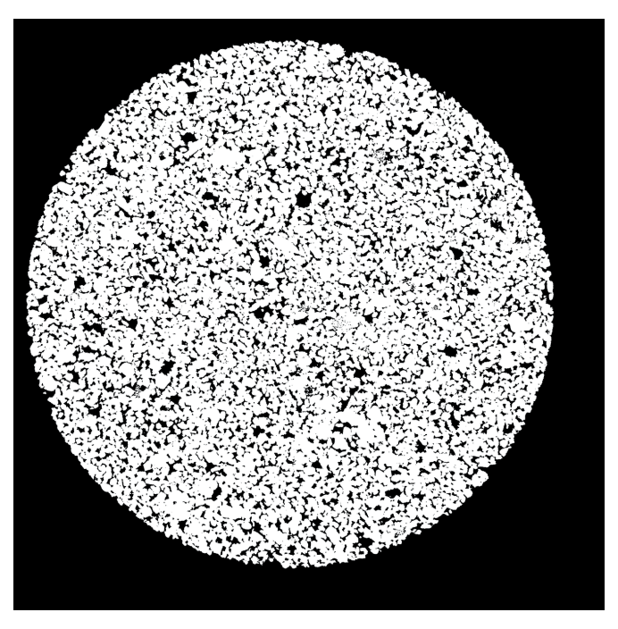

Fig. 1: 2D cross-section of binary data 2240 x 2240 pixels

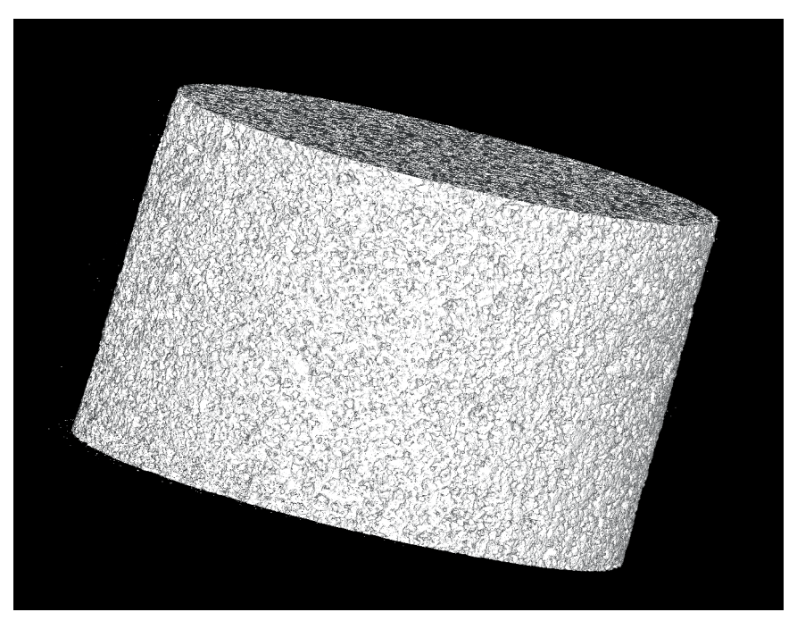

Fig. 2: 3D binary image 1288 x 2240 x 2240 voxels

Distance-Transform PH (via HomCloud)[2]**:** We also apply HomCloud, a persistent homology software optimized for image data. HomCloud uses a signed distance transform on the binary image to create a filtration: one phase (e.g. void space) is gradually "grown" or thickened, and the algorithm tracks the creation and filling of holes. This captures connected components and holes directly on the voxel grid. HomCloud is an open-source tool focused on image-based PH, known to be fast and scalable for large 3D voxel datasets.

Results

Normalization & Metrics: Betti numbers are evaluated to quantify different topological features: B0 represents the number of connected components, B1 corresponds to the number of loops, and B2 represents the number of cavities. Once we had persistence outputs for a given cube, we counted the number of finite persistence pairs (i.e. the number of topological features, or 'generators') in each homology dimension (B0, B1, B2) and summed their lifespans to obtain total persistence .

These raw counts and sums were then normalized by the cube's volume (V), which is the total number of voxels in that cube L^3 to yield *#Generators/V* and *Total Persistence/V*. This per-voxel normalization (i.e. densities) is critical for comparison across different cube sizes.

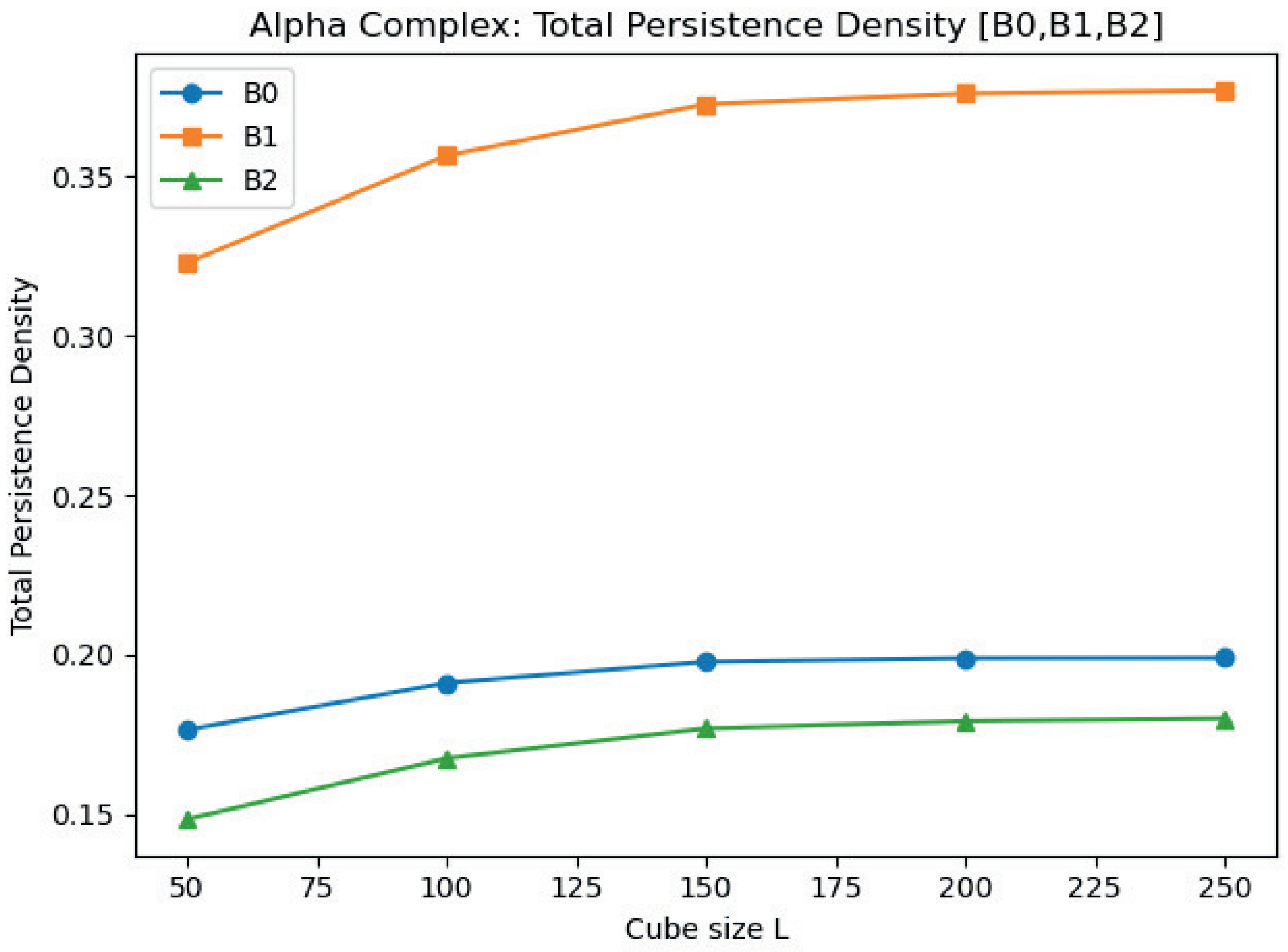

Fig. 3 Total Persistence Density vs Cube Size using Alpha Complexes for overlapping cubes

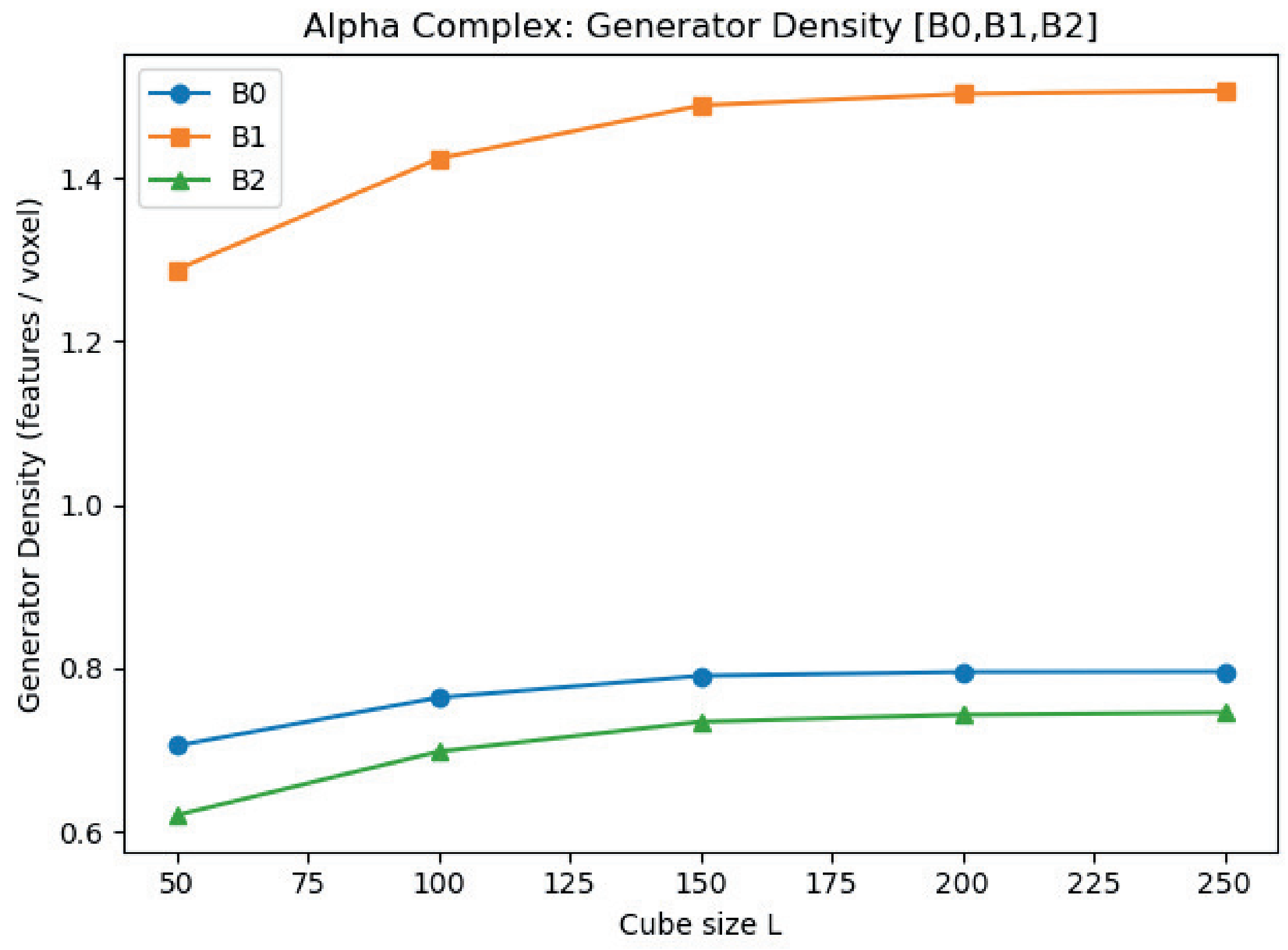

Fig. 4 #Generator Density vs Cube Size using Alpha Complexes for overlapping cubes (Here 'generator' means one persistent topological feature i.e. one point in the persistence diagram)

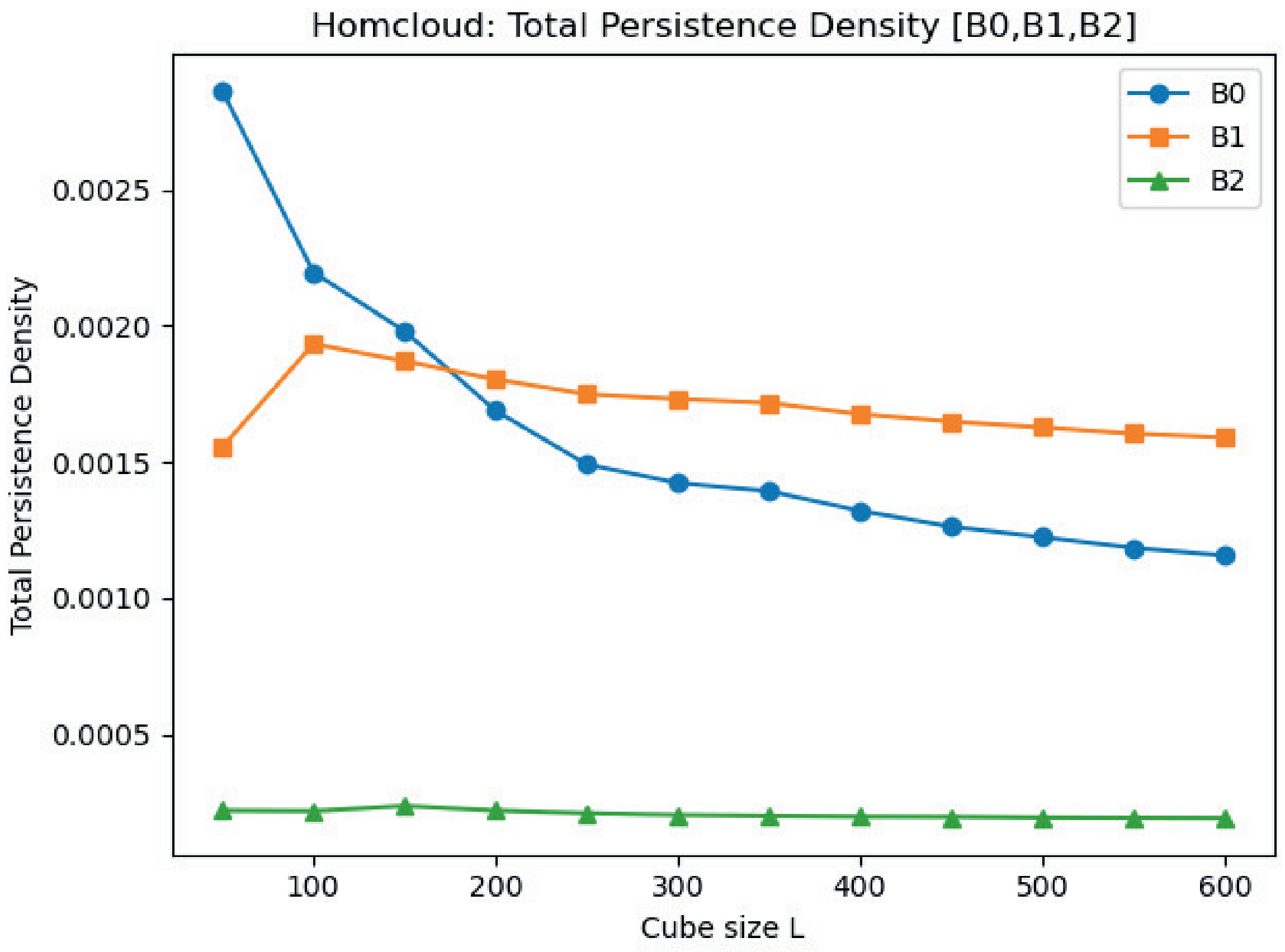

Fig. 5: Total Persistence Density vs Cube Size using Homcloud's Distance Transform for overlapping cubes

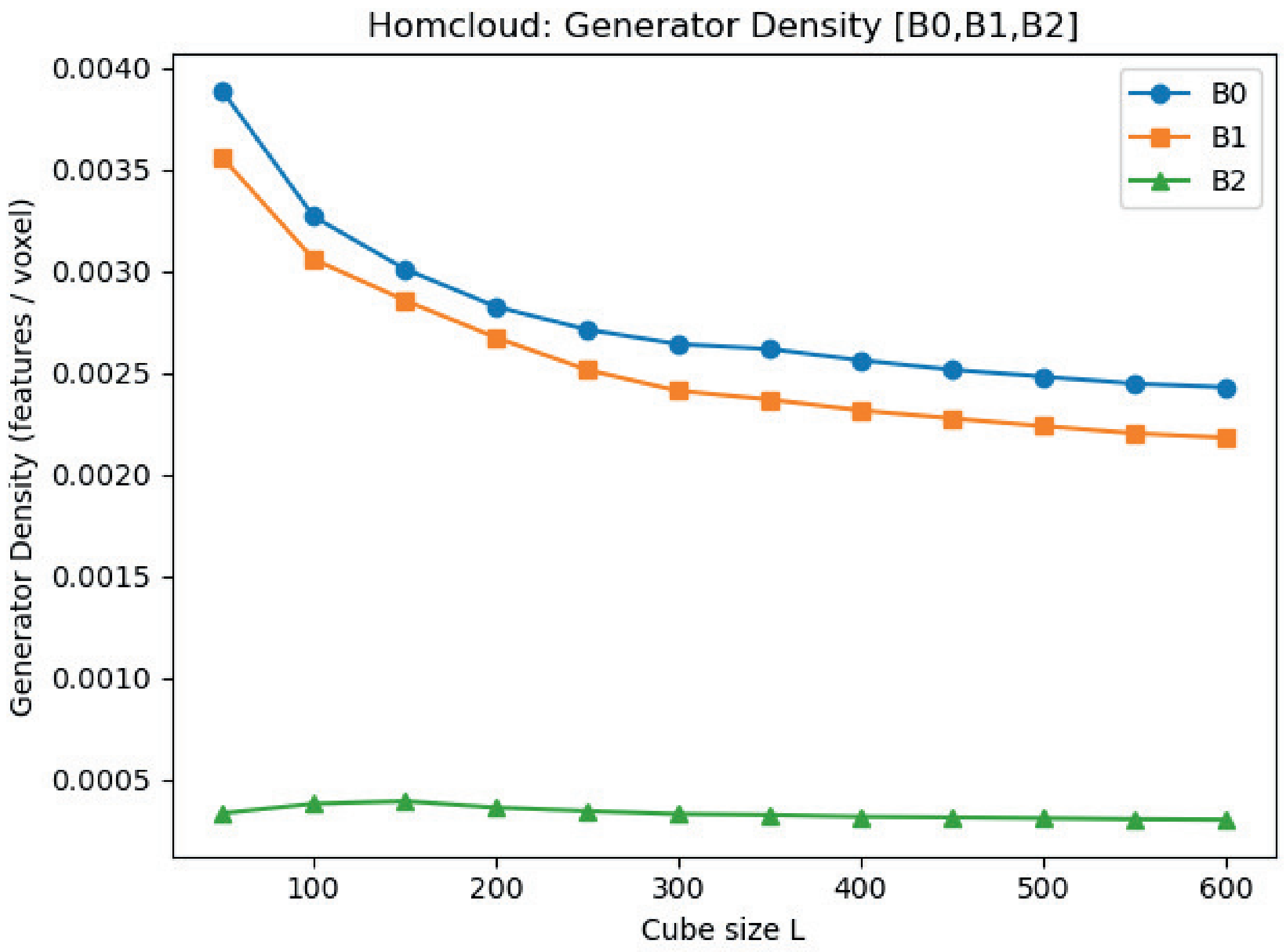

Fig. 6 Generator Density vs Cube Size using Homcloud for overlapping cubes

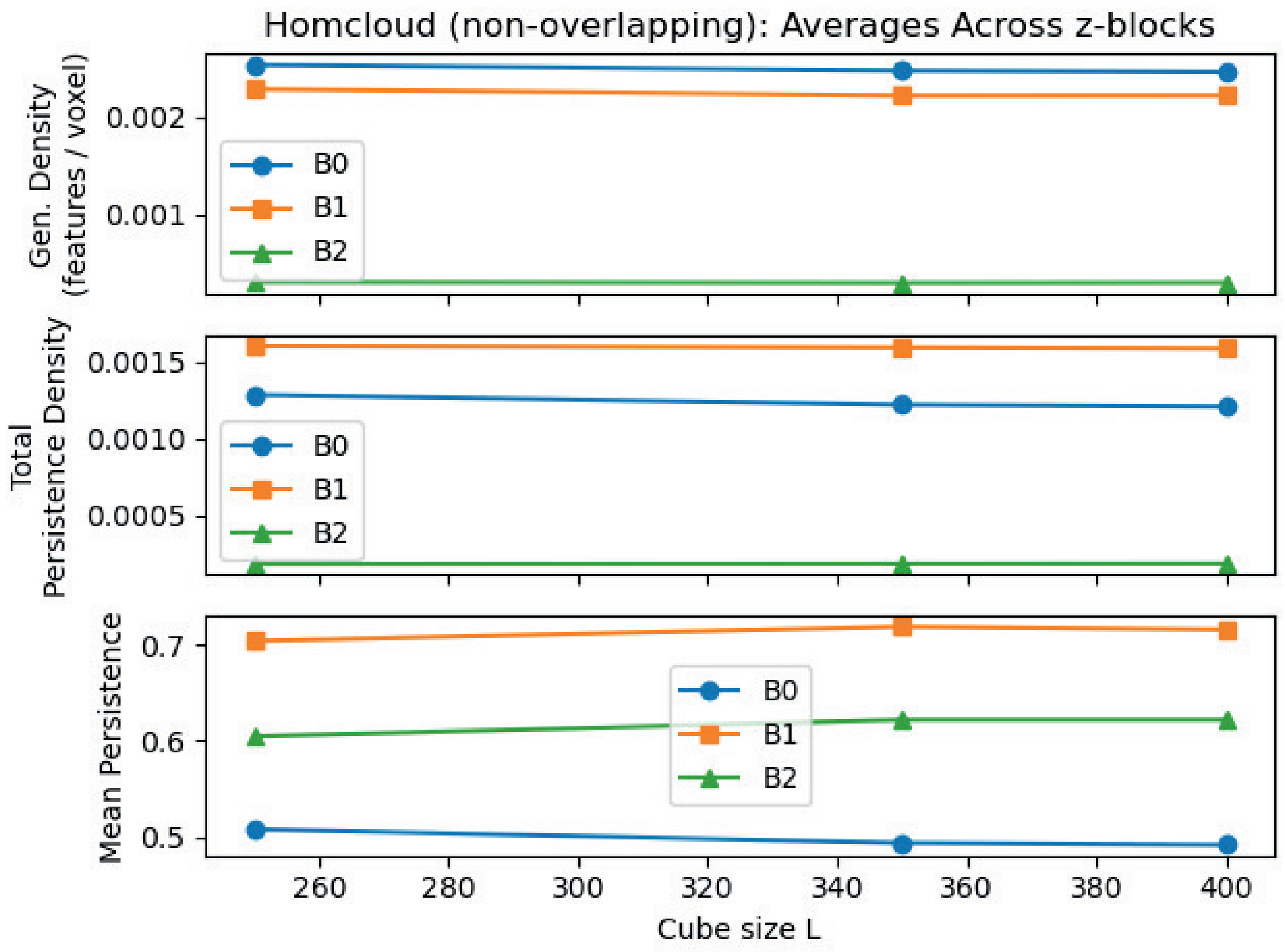

Fig. 7 Generator Density,Total Persistence Density, Mean Persistence (Avg. Lifespan) vs Cube Size using Homcloud for non-overlapping cubes. Each 'z-block' is a separate cube taken from a different depth range along the sample's z-direction (for example z = 0–250, 250–500, etc.); these cubes do not overlap.

Summary

- **Convergence of Topological Metrics:** Both the number of features per volume and the total persistence per volume converge to constant values as the cube size increases. By around a 200^3 voxel subdomain, the generator density and persistence density stop changing appreciably with further size, indicating a representative elementary volume for topology has been reached.
- **Consistency Across Locations:** Separate large subdomains taken from different locations in the material exhibit nearly identical topological measures.
- **Homogeneity Confirmed:** The agreement between two very different PH approaches (alpha complexes vs. distance-based) and the stabilization of all measured metrics strongly support that the material is topologically homogeneous.

References

[1] The GUDHI Project. (2025). GUDHI User and Reference Manual (Edition 3.11.0). GUDHI Editorial Board.
[2] Suzuki, A., etal. (2021). Flow estimation solely from image data through persistent homology analysis. Scientific Reports, 11, 17948.
[3] Data courtesy of Dr. E. Domene, YTEC, Argentina