Stability and collapse of holes in liquid layers with vibrations

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Abstract

We investigate experimentally and theoretically the stability and collapse of holes in liquid layers on bounded substrates with the addition of surface vibrations.

It is shown that for a liquid layer with a thickness of the order of the capillary length, a stable hole exists when the hole diameter is bigger than a critical value d_c.

Consequently, a further increase of the liquid volume causes the hole to collapse. It is found that d_c increases with the size of the container, but decreases with the magnitude of the surface vibrations [1].

Motivation

Control of the evolution of vibrated wetting fluids on the micro-scale. Example: Smoothly adding liquid to a vibrated film containing a hole until an unstable state is reached [1].

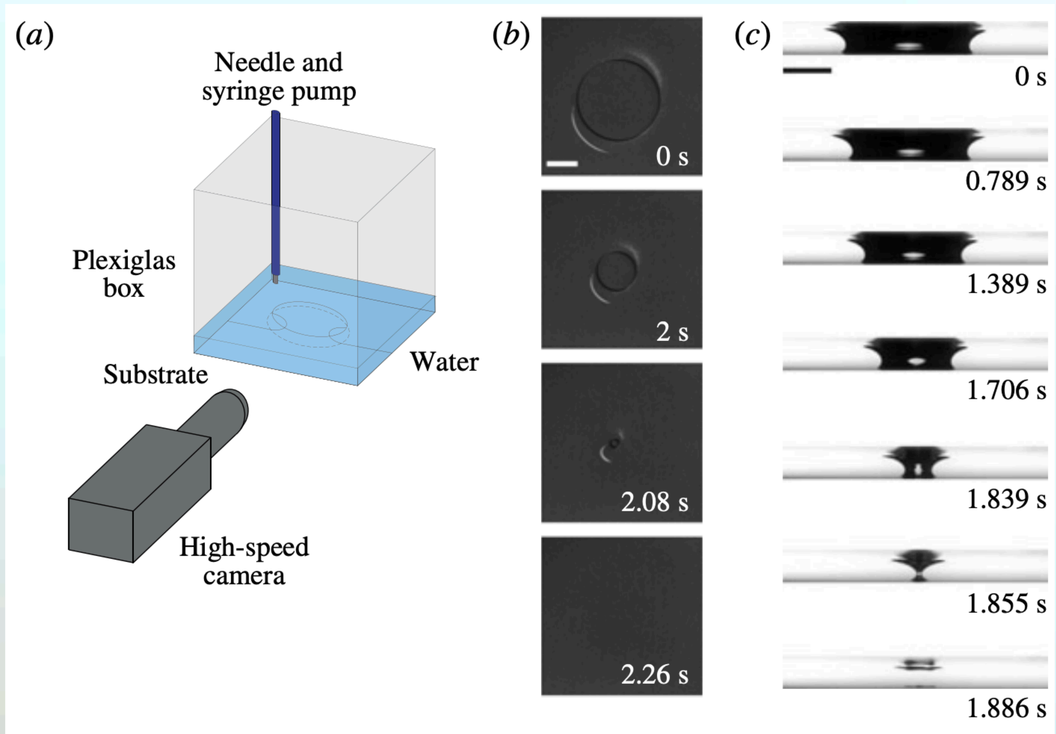

Fig. 1: (a) Schematic illustration of the experimental set-up with a needle connected to a syringe pump controlling the flow rate. Time lapse images of the collapse of a hole at the centre of a liquid layer on a bounded superhydrophobic Al plate are shown in top (b) and side (c) view, which are results from two different experiments.

Main assumptions

- Long wave theory describing thin film dynamics is used [2].
  - Direction normal to the plane must be shorter than any in plane dimensions
  - The slope of the free surface is small
- We are working in an axis-symmetric coordinate system
- Neumann conditions were used at the left and right boundary along with zero flux condition on the left but fixed non-zero flux on the right

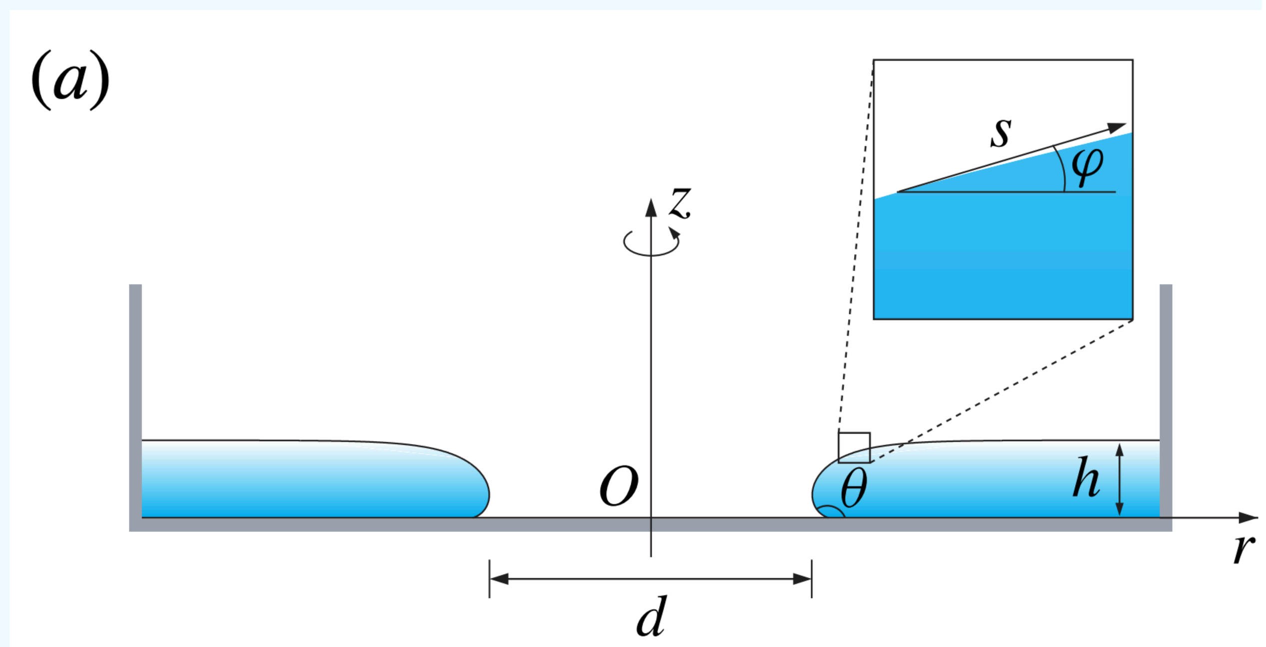

Fig. 2: Sketch of the modeling problem.

Equations

$$\frac{\partial h}{\partial t} = \frac{1}{r}\left[rh^3\left((1+\hat{C}\cos(\omega t))h_r - Kf'h_r - T_r\right)\right]_r$$

$$\hat{C} = \frac{\overline{C}}{g} = \frac{A\omega^2}{g} = \frac{4\pi^2 A f^2}{g}$$

$$K = \frac{\kappa a}{\gamma} = \frac{(1-\cos(\theta))(m-1)(n-1)}{h_*(n-m)}$$

$$f(h) = \left(\frac{h_*}{h}\right)^n - \left(\frac{h_*}{h}\right)^m$$

$$T = \frac{1}{r}(rh_r)_r$$

$$x_c = \left(\frac{\gamma h_c}{\rho g}\right)^{1/3} \qquad t_c = \frac{3\mu x_c^4}{\gamma h_c^3}$$

Results

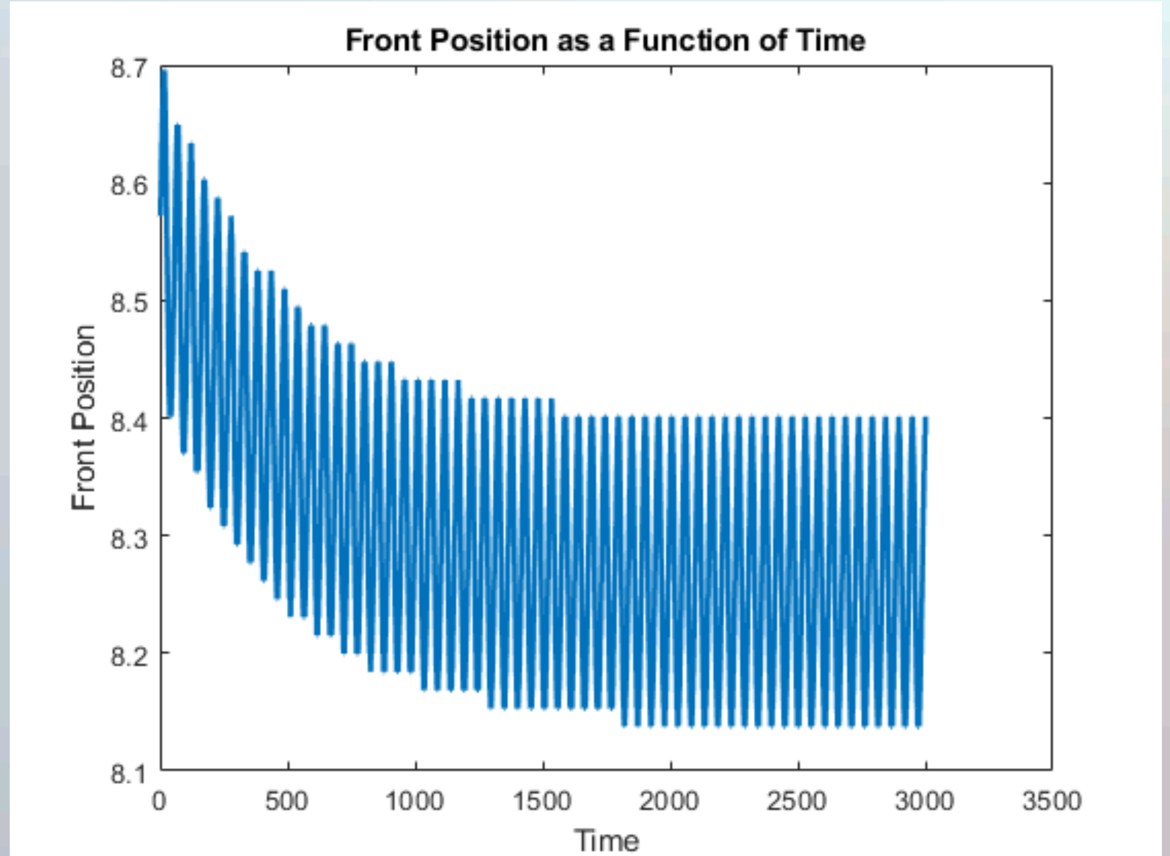

Fig. 3: Fluid front evolution as a function of time for $\hat{C}$ = -2

The fluid front oscillates with the same frequency as the surface vibrations and approaches a new stable radius size as time progresses.

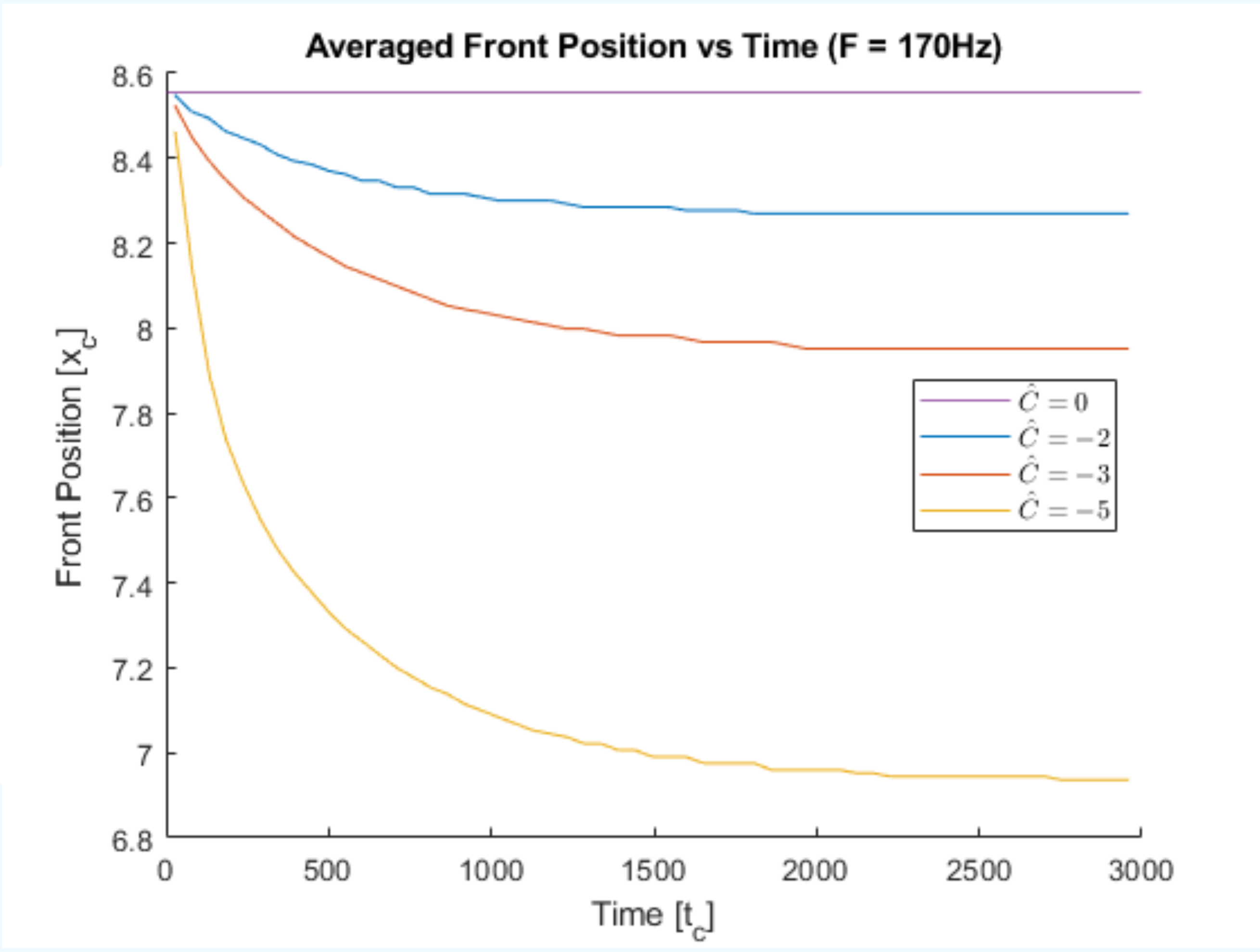

Fig. 4: Averaged front position as a function of time for decreasing $\hat{C}$ values

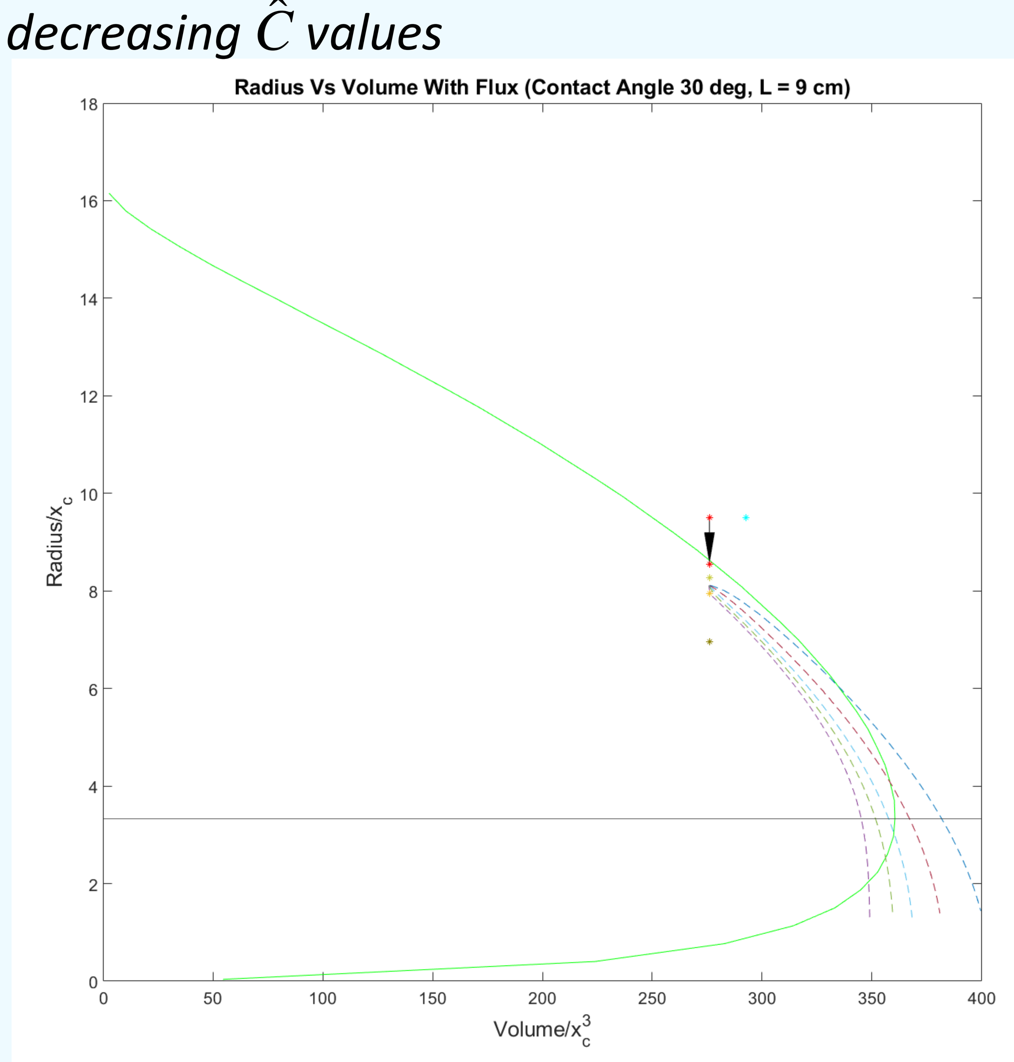

Fig. 5: Fluid front stability plot for $\hat{C}$ = -3 and decreasing values of supplied flux

Summary

Vertical surface vibrations can cause a decrease in the stable hole size of holes in liquid layers.

As flux is supplied at decreasing rates we can find the new critical diameter for a given value of $\hat{C}$.

This new d_c is shown to be less than the critical diameter without vibrations.

Future Work: Experiments involve larger values of Reynolds number so we would like to include inertial terms in our simulations.

References

[1] Lv, Cunjing, et al. "Stability and Collapse of Holes in Liquid Layers." Journal of Fluid Mechanics, vol. 855, 2018, pp. 1130–1155.

[2] Kondic, L. "Instabilities in Gravity Driven Flow of Thin Fluid Films." SIAM Review, vol. 45, no. 1, 2003, pp. 95–115.