Nonlinear Dynamics of Vibrating Fluid Drops

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Abstract

We investigate experimentally and theoretically the spreading and deformation of fluid puddles on bounded substrates under vertical surface vibrations.

For a fluid volume with a characteristic height on the order of the capillary length, a puddle of partially-wetting liquid remains stable in the absence of forcing. When vibrated vertically, the puddle front expands progressively, with the extent of spreading increasing with vibration strength.

This behavior is captured by an effective capillary length framework, where vibrational forcing reduces the apparent surface tension of the fluid [1].

Motivation

Controlling the spreading of liquid films through vibration offers a non-invasive way to control fluid spreading in applications like coating, enabling tunable control of contact line motion through effective surface forces, without modifying surface chemistry.

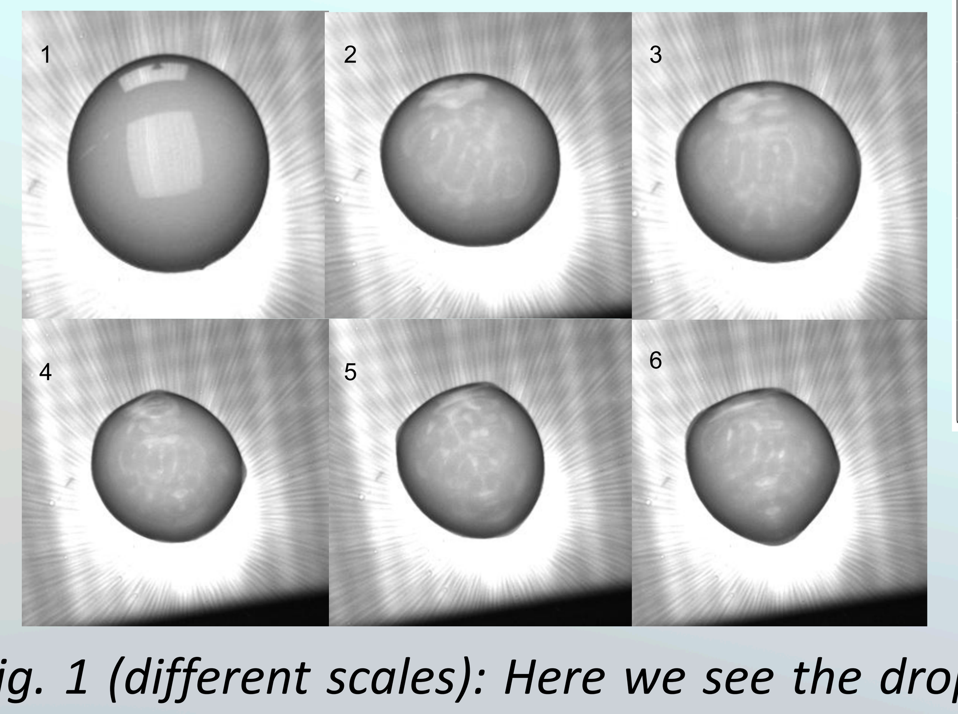

Fig. 1 (different scales): Here we see the drop from above when applying different excitation amplitudes. Starting with no oscillations in the top left (1) and 60% of the max amplitude in the top row, middle column (2). Then as we move right (3)-(6) the amplitude increases by 10% of the max value until we reach 100% in the lower right.

Main Assumptions

- The use a thin-film approach that includes disjoining pressure, contact angle dynamics, vibrational acceleration, and assumes axisymmetry [2].
- Neumann conditions imposed at drop boundary, along with zero flux condition.

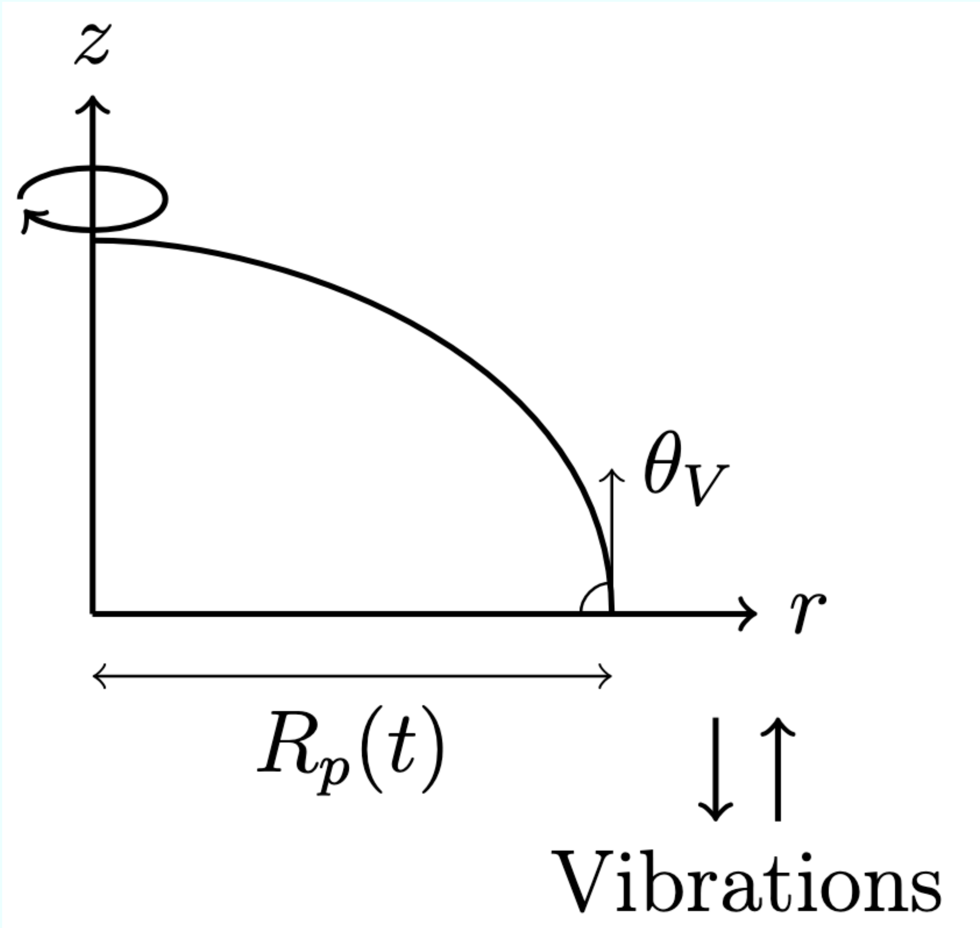

Fig. 2: Sketch of the modeling problem.

Equations

$$\mathcal{M}\frac{\partial h}{\partial t} = \frac{1}{3r}\frac{d}{dr}\left[rh^3\left((1+\hat{C}\cos(\omega t))h_r - \Gamma(Kf_p'(h)h_r + T_r)\right)\right]$$

$$\mathcal{M} = \frac{\mu}{\mu_0} \qquad \Gamma = \frac{\gamma}{\gamma_0} \qquad \hat{C} = \frac{\overline{C}}{g} = \frac{A\omega^2}{g} = \frac{4\pi^2 A f^2}{g}$$

$$f_p(h) = \left(\frac{h_p}{h}\right)^n - \left(\frac{h_p}{h}\right)^m \qquad K = \frac{\kappa a}{\gamma} = \frac{(1-\cos(\theta))(m-1)(n-1)}{h_*(n-m)}$$

$$T = \frac{1}{r}(rh_r)_r \qquad x_c = \sqrt{\frac{\gamma_0}{\rho g}} \qquad t_c = \frac{\mu_0 x_c}{\gamma_0} \qquad p_c = \frac{\gamma_0}{x_c}$$

Variable and Parameter Definitions

Symbol	Physical Meaning	Symbol	Physical Meaning
h	Fluid Height	μ	Variable Viscosity
A	Amplitude of Vibrations	μ_0	Dynamic Viscosity of water
ρ	Density of water	γ	Variable Surface Tension
g	Gravity	γ_0	Surface Tension of water
f	Frequency of Vibrations	$\hat{C}$	Non-dimensional Acceleration of Vibrations
h_p	Precursor Film Thickness	θ	Contact Angle
R_p^{eq}	Static Equilibrium Front Position	R_{pv}^{eq}	Dynamic Equilibrium Front Position
$\mathcal{M}$	Ratio of Variable Viscosity to Fixed	Γ	Ratio of Variable Surface Tension to Fixed
x_c	Length Scale	t_c	Time Scale

Results

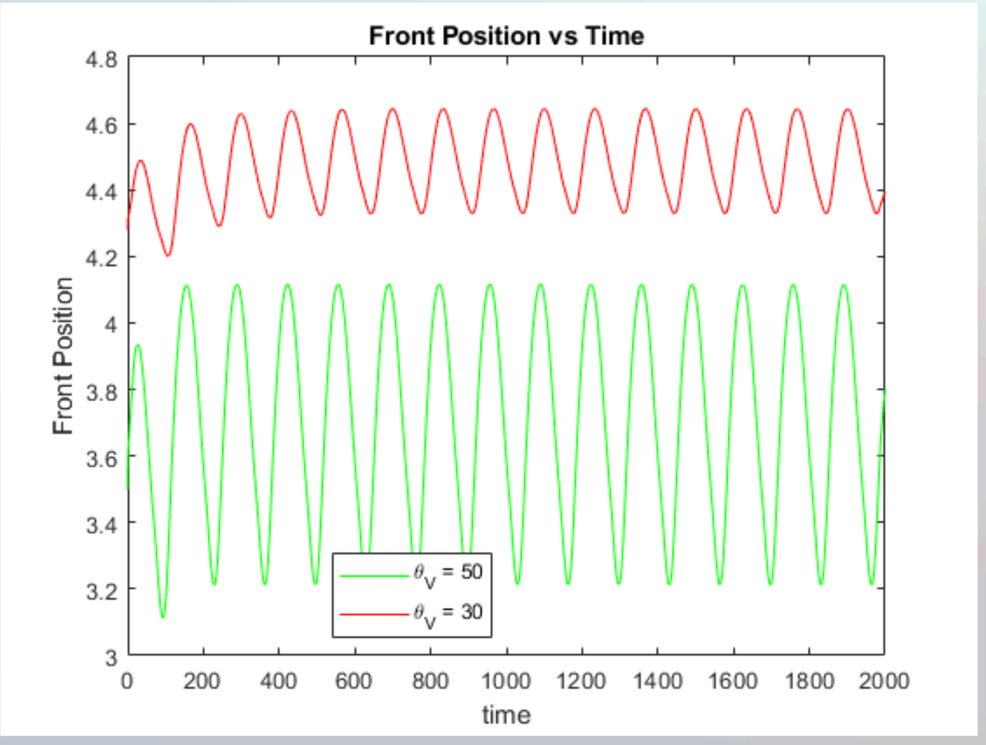

Fig. 3: Fluid front evolution as a function of time for $\hat{C}$ = 4.

After the initially confined puddle reaches equilibrium, vibrations are applied and the front position is tracked. The fluid front oscillates with the same frequency as the surface vibrations and approaches a new stable radius size as time progresses.

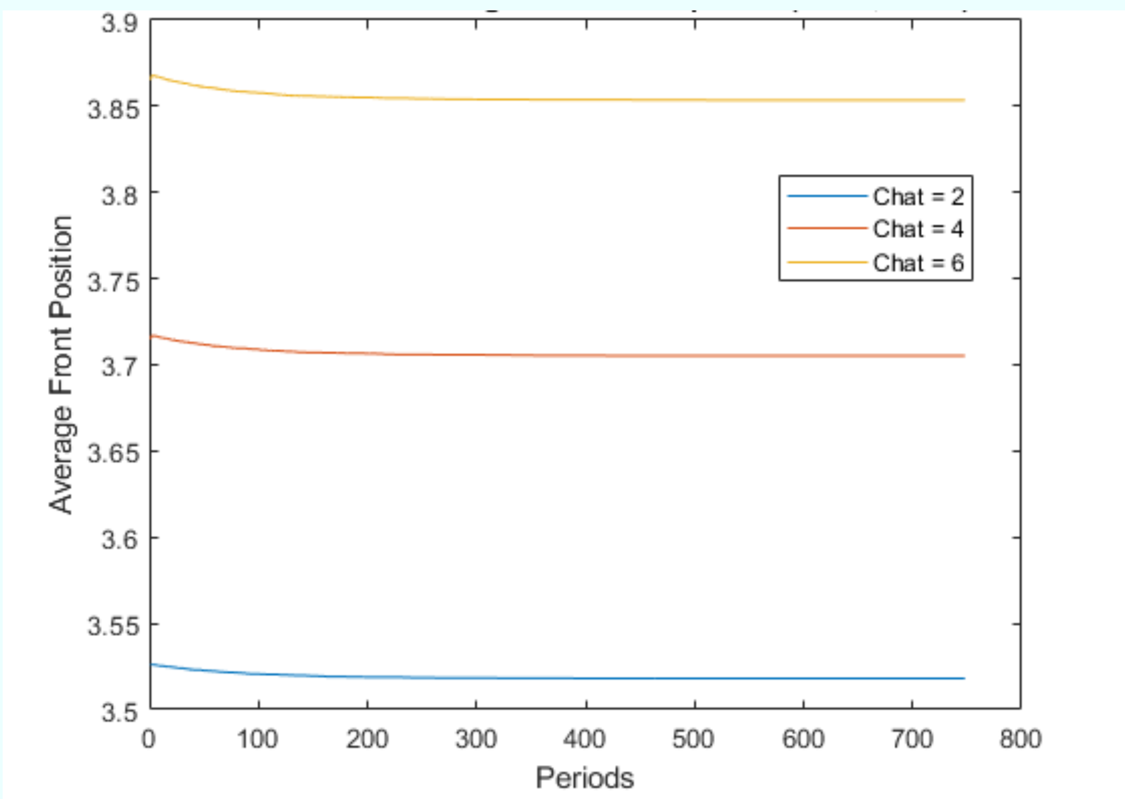

Fig. 4: Front position averaged over each period for different values of $\hat{C}$ with M = 1 and Γ = 1.

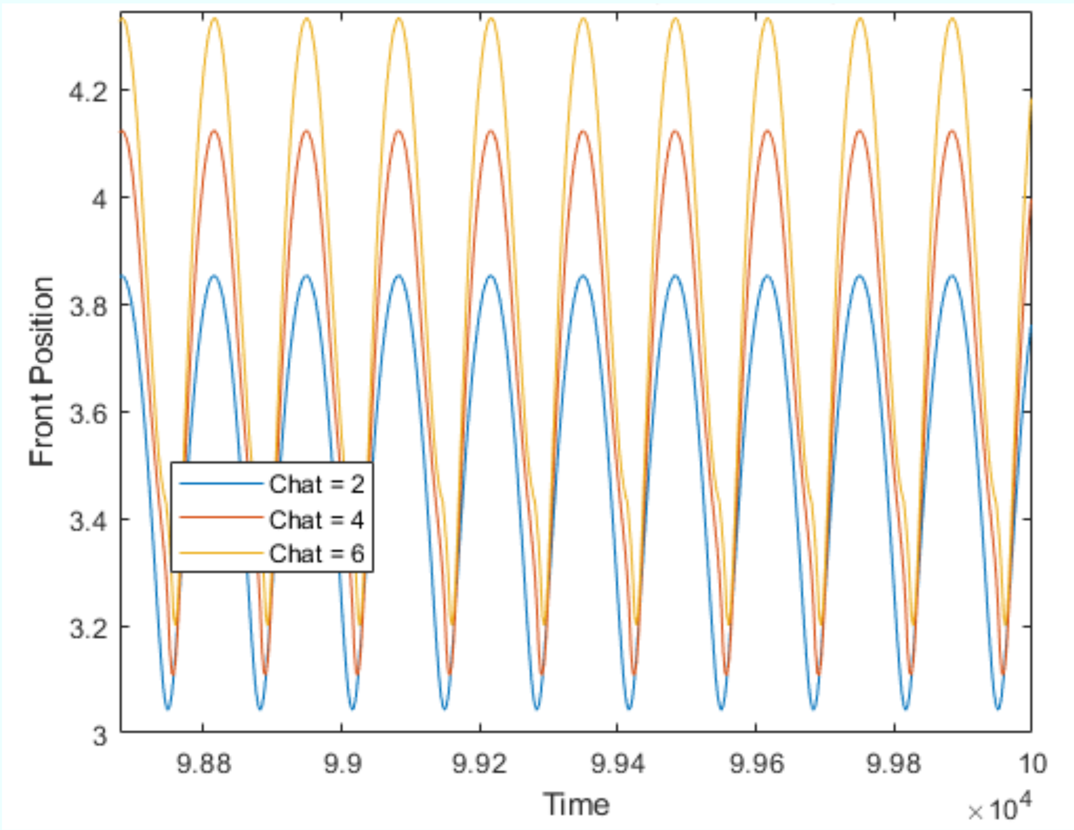

Fig. 5: Front position vs time (last 10 periods) for different values of $\hat{C}$ with M = 1 and Γ = 1.

Summary

Vertical surface vibrations can significantly modify the steady-state shape of fluid puddles on solid substrates. Under vibrational forcing, the steady-state front position R_{pv}^{eq} increases compared to the static case R_p^{eq}, indicating outward spreading of the fluid.

We analyze this behavior using a thin-film model, varying both the vibrational forcing strength $\hat{C}$ and the contact angle θ. Simulations show that higher $\hat{C}$ values enhance the spreading of the puddle, increasing R_{pv}^{eq}. Additionally, smaller contact angles θ also lead to more pronounced spreading.

This approach quantitatively links vibrational forcing and contact angle to the steady-state front position, capturing how vibrations and wetting conditions shape the equilibrium puddle configuration.

Future work includes extending the model to include inertial effects for higher Reynolds number cases.

References

[1] Bisswanger, Steffen, et al. *Physical Review Letters*, vol. 133, 034001 (2024).

[2] Kondic, L. SIAM Review, vol. 45, 95 (2003).