

Abstract

We investigate experimentally and theoretically spreading of a millimetric silicon oil drop under the excitation of a surface acoustic wave (SAW) propagating through the underlying substrate.

The acoustic force is related to Eckart streaming, a mechanism resulting from variations in the Reynolds's stress in the liquid, and in the intensity of ultrasonic wave leakage off the attenuating SAW under the drop. For the highly viscous oils considered in our experiments, the effective SAW force is also due to viscous dissipation in the liquid. [1]

Our model provides good qualitative predictions for drop shapes, spreading speeds, and the dependence of the results on the SAW displacement amplitude at the solid surface (the SAW intensity).

Motivation

SAWs have demonstrated the potential to separate oil-water emulsions at small scales [2], suggesting a possible application to graywater recycling. To begin to understand the mechanisms by which this separation occurs, we first study the single-phase oil droplets to develop a theoretical framework for their dynamics.

The single-phase problem can also be extended to include topographical features on the solid substrate. Coating over topography is widely used in the fabrication of microelectronic components, integrated circuits, manufacturing of magnetic memory devices, and magnetic disks, compact disks, and optical devices.

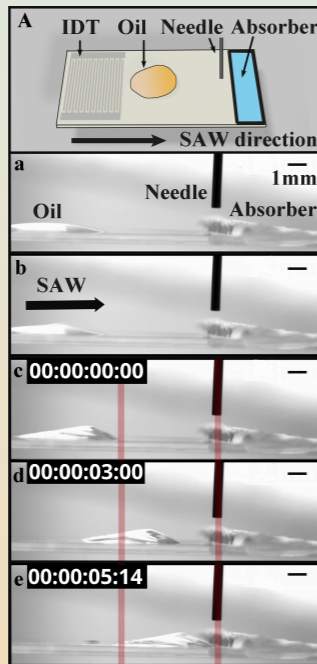


Fig. 1: (A) Upper schematic view of the experimental setup showing direction of SAW propagation. SAW propagates from the interdigitated transducer (IDT) until it reaches the acoustic absorber. (a-e) Snapshots of drop evolution at stated times, showing steady translation.

Assumptions & Background

- Assume that the associated acoustic Mach number, $M_a \equiv \omega A / c$, is small and that the velocity (v), pressure (p), and density (ρ) fields can be expanded in terms of M_a [3,4]
- Here, A is the maximum vertical displacement of the substrate caused by the SAW, ω is the frequency of the SAW, K_z is the attenuation factor of the wave under silicon oil in the z -direction, and C_z is a coefficient of the z -component of the SAW force.
- Further, ρ_0 is the constant density of the silicon oil, g is the gravitational acceleration, and μ is the viscosity
- The gravitational contribution appears at the second-order in M_a
- Assume translational invariance in the y -direction (2D model)
- Use long-wave theory to simplify the second-order system and derive a single PDE for the height of the film, $h(x, t)$

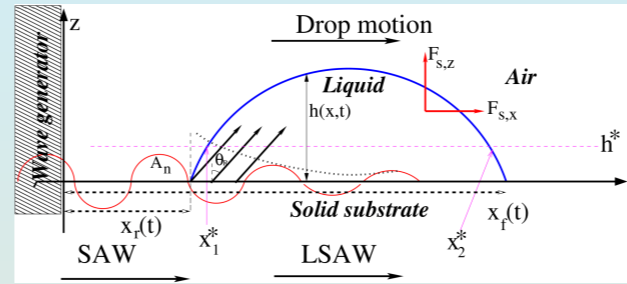


Fig. 2: Sketch of the physical problem. A drop of silicon oil atop a solid substrate, through which a SAW propagates. The SAW experiences negligible attenuation under air and significant attenuation under the bulk fluid.

Model Equations

The second-order problem, by time-averaging over the period of the SAW [4], can be reduced to a Stokes flow with an external acoustic force:

$$-\nabla \langle p_2 \rangle + \mathbf{F}_s - \rho_0 g \mathbf{e}_z + \mu \nabla^2 \langle \mathbf{v}_2 \rangle = 0,$$

$$\nabla \cdot \langle \mathbf{v}_2 \rangle = 0,$$

$$\mathbf{F}_s = -\langle \rho_0 (\mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + \mathbf{v}_1 \nabla \cdot (\rho_0 \mathbf{v}_1) \rangle.$$

Using the standard long-wave expansion we can reduce this to a single PDE for the height, $h(x, t)$ [5]:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \left[-h^3 \frac{\partial \mathcal{P}}{\partial x} - \mathcal{E} \frac{3\mathcal{S}}{8K_z^2} \psi(x, h) (2K_z^2 h^2 - 1 + e^{2K_z h} (1 - 2K_z h)) \right] = 0,$$

$$\mathcal{P} = -\frac{\partial^2 h}{\partial x^2} + \text{Bo} h + \frac{\mathcal{S} C_z}{2K_z} \psi(x, h),$$

where we defined the non dimensional parameters

$$\text{Bo} = \frac{\rho_0 g \ell^2}{\gamma} = \frac{\ell^2}{a^2}, \quad \mathcal{S} = \frac{\rho_0 \ell A^2 \omega^2}{\gamma}, \quad \mathcal{E} = K_z C_x - k_{s,i} C_z$$

Numerical Results

For low SAW amplitudes ($A < 2\text{nm}$), capillary and gravitational forces dominate: for $A=1.3\text{nm}$, the drop spreads in both directions albeit the acoustic force leads to faster spreading in the direction of the propagating wave (Fig 3A). As the SAW amplitude increases, both contact lines begin to move in the same direction (Fig 3B, 3C). As the drop translates, a thin film is left behind (Fig 3C), similar to that seen in experiments; this causes the maximum height of the drop to slightly decrease over time. For large SAW amplitudes a traveling wave profile develops at long times (Fig 3C).

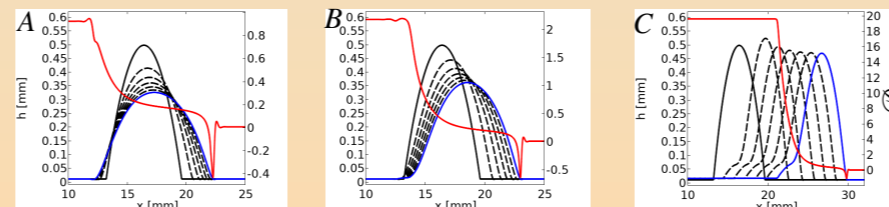
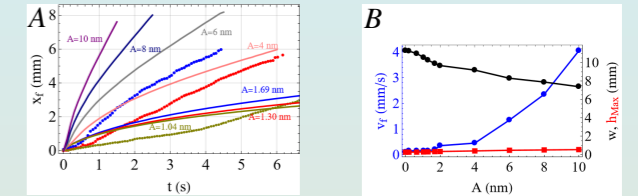


Fig. 3: Initial height profile (solid black), intermediate height profiles every $\Delta t = 1\text{s}$ (dashed black), final height profile at $t = 6\text{s}$ (solid blue), and the final effective pressure \mathcal{P} (solid red). Frame A-D correspond to SAW amplitudes of $A = 1.3, 2, 10\text{ nm}$ respectively.

Fig. 5: (A) Front contact line position as a function of time for various SAW amplitudes. The solid lines denote simulations and the dots denote experimental results. (B) Drop width, w , front speed of translation, v_f , and maximum height of the drop, h_{Max} , for various values of A .



In view of these results, we explore a possible traveling wave solution with speed U [5]. We define $\xi = x - Ut$ and transform the PDE to an ODE for $h(\xi)$:

$$U h + h^3 \frac{d}{d\xi} \left[-\frac{d^2 h}{d\xi^2} + \text{Bo} h + \frac{\mathcal{S} C_z}{2K_z} \psi(\xi, h) \right] + \mathcal{E} \frac{3\mathcal{S}}{8K_z^2} \psi(\xi, h) (2K_z^2 h^2 - 1 + e^{2K_z h} (1 - 2K_z h)) = J.$$

The solution must be calculated for $0 \leq \xi \leq w$ with the boundary conditions

$$h(0) = h^*, \quad h'(0) = h''(0) = 0,$$

The values of $h''(0)$, U , w are determined by

$$h(w) = h^*, \quad J(0) = J(w), \quad A_d = \int_0^w h d\xi.$$

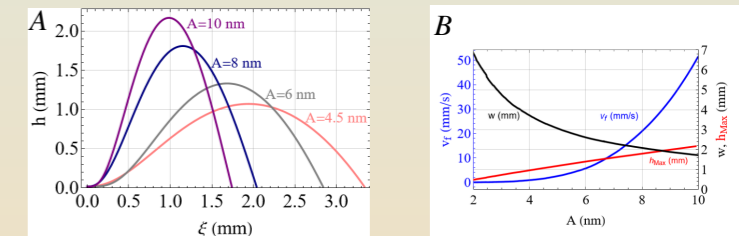


Fig. 6: (A) Traveling wave height profiles. (B) Corresponding values of the drop width, speed of translation, and maximum height for various A -values.

Conclusions

Simulation results provide qualitative agreement between theoretical model and experiments. In contrast to the familiar case of gravity driven flow, where the force is constant and not rapidly decreasing along the drop, here we see near constant driving speed and drop shape at long times for sufficiently large SAW amplitudes.

This finding motivated us to formulate a simplified traveling wave model. The resulting analytical solution predicts an approximately linear increase in the drop thickness and a quadratic increase of the spreading speed with respect to the SAW amplitude. Such predicted trends are consistent with the full theoretical model and experiments.

References

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