Ivana Seric, Shahriar Afkhami, Lou Kondic
Department of Mathematical Sciences and Center for Applied Mathematics and Statistics, NJIT, Newark, NJ

Abstract

The volume of fluid (VOF) interface tracking methods have been used for simulating a wide range of interfacial flows. An improved accuracy of the surface tension force computation has enabled the VOF method to become widely used for simulating surface tension driven flows. We present a new method for including variable surface tension in a VOF based Navier-Stokes solver. The tangential gradient of the surface tension is implemented using an extension of the classical continuum surface force model that has been previously used for constant surface tension simulations. Our method can be used for computing the surface gradients of surface tension that is temperature or concentration dependent [? ?].

Introduction

The flow is governed by two-phase incompressible Navier-Stokes equations

$$\rho(\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}) = -\nabla p + \nabla\cdot\left(\mu\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)\right) + \sigma\kappa\delta_s\hat{\mathbf{n}} + \frac{\partial\sigma}{\partial s}\delta_s\hat{\mathbf{t}} \quad (1)$$

$$\nabla\cdot\mathbf{u} = 0$$

- $\mathbf{u} = (u, v, w)$ is the fluid velocity, p is the pressure
- $\rho(\chi) = \chi\rho_1 + (1-\chi)\rho_2$ and $\mu(\chi) = \chi\mu_1 + (1-\chi)\mu_2$ are the phase dependent density and viscosity respectively
- χ is the volume fraction function, where $\chi = 1$ in the fluid, and $\chi = 0$ in the surrounding, and it is advected with the flow, i.e. $\partial_t\chi + \nabla\cdot(\mathbf{u}\chi) = 0$
- The last two terms correspond to the surface tension force which is a result of the stress boundary condition at the interface, $\mathbf{F}_s = \sigma\kappa\hat{\mathbf{n}} + \nabla_s\sigma$
- The surface tension force is included as a volume force using the continuum surface force method (CSF) [?].
- The surface tension is a function of temperature

$$\sigma = \sigma_0 + \sigma_T(T - T_0) \quad (2)$$

- The temperature satisfies advection-diffusion equation

$$\rho C_p\left[\partial_t T + \nabla\cdot(\mathbf{u}T)\right] = \nabla\cdot(k\nabla T) \quad (3)$$

Numerical Method

The tangential gradient of the surface tension is implemented using Gerris free solver [? ?]. In the case of The surface tension coefficient is a linear function of temperature, can write $\partial\sigma/\partial s = \sigma_T\partial T/\partial s$. Here, we describe the algorithm for computing $\partial T/\partial s$. The same method can be applied if σ is a nonlinear function of temperature or concentration, where we would compute $\partial\sigma/\partial s$ in the same manner at $\partial T/\partial s$.
The algorithm can be divided into two parts:

1. Approximating interfacial temperature;
2. Computing the surface gradient.

Acknowledgment

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Interfacial Temperature

The temperature of the interface $\tilde{T}$ is approximated using the values of the temperature field T in the interfacial cells, so that $\tilde{T}$ has one value in each column. For columns with only one interfacial cell the temperature of the interfacial cells $\tilde{T}$ is equal to the temperature T in the same cells. If there is more than one interfacial cell in the column, then the volume weighted average is used for approximating the interfacial temperature $\tilde{T}$.

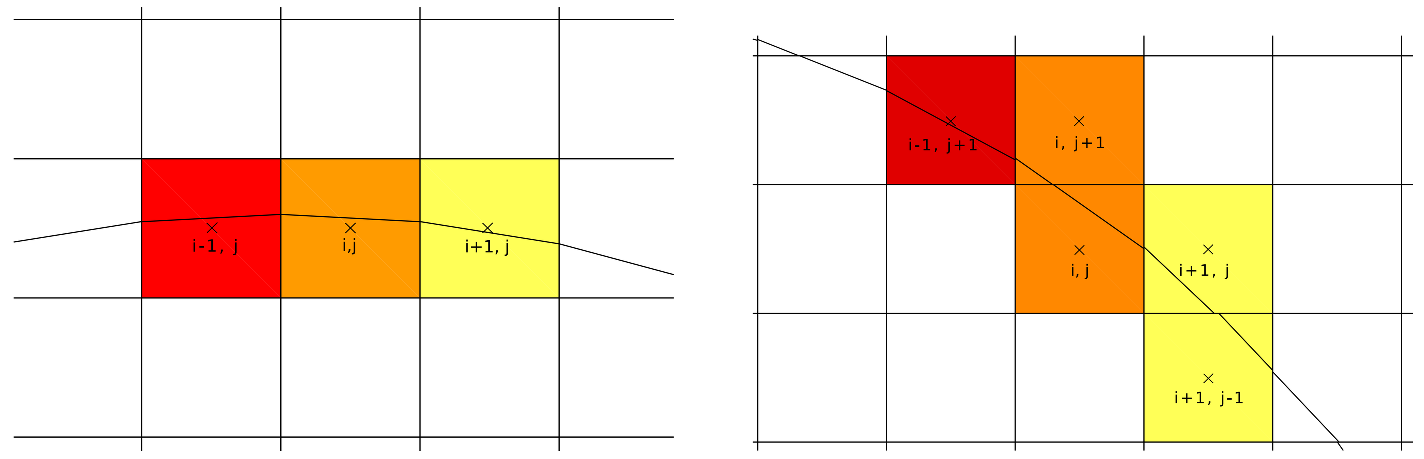

Figure 1: Examples of interface geometry where the interfacial temperature $\tilde{T}$ is computed using the columns in the y direction. E.g. the interface temperature in $\mathcal{C}_{i,j}$ on the right is

$$\tilde{T}_{i,j} = \frac{\chi_{i,j}T_{i,j} + \chi_{i+1,j}T_{i+1,j}}{\Sigma\chi_i}.$$

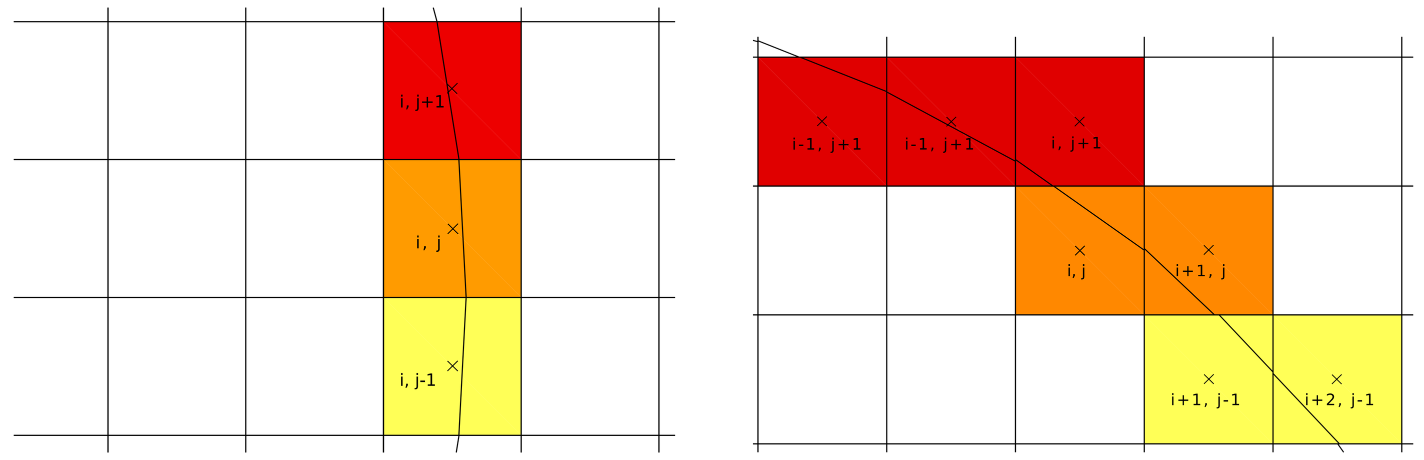

Figure 2: Examples of interface geometry where the interfacial temperature $\tilde{T}$ is computed using the columns in the x direction. E.g. the temperature of the cell $\mathcal{C}_{i,j}$ on the right is

$$\tilde{T}_{i,j} = \frac{\chi_{i,j}T_{i,j} + \chi_{i,j+1}T_{i,j+1}}{\Sigma\chi_i}.$$

Surface Gradient

Gradients can be computed using the columns in either x or y directions, depending on the orientation of the interface; we chose the direction of the largest component of the normal vector at the interface. For example in Figure 2(b), the gradient is calculated in the y direction as

$$\left(\frac{\partial T}{\partial s_y}\right)_{i,j} = \frac{\tilde{T}_{i+1,j} - \tilde{T}_{i-1,j+1}}{\mathrm{d}s}. \quad (4)$$

The arc length, $\mathrm{d}s$, is computed from the height function in the same direction as $\nabla_s T$. In the same example as above, the arc length is

$$\mathrm{d}s = 2\Delta\sqrt{1 + h_y}, \quad (5)$$

where h_y is the derivative of the height function [?].

Results

We test the accuracy of our method by comparing with the existing literature for a classical test case of a thermocapillary migration of a drop or radius a, placed in an ambient fluid with imposed uniform temperature gradient $|\nabla T_\infty|$. The nondimensional parameters governing the motion are

$$\mathrm{Re} = \frac{\rho_0\sigma_T|\nabla T_\infty|a^2}{\mu_0^2}, \quad \mathrm{Ca} = \frac{\sigma_T|\nabla T_\infty|a}{\sigma_0}, \quad \mathrm{Ma} = \frac{\rho_0 C_{p0}\sigma_T|\nabla T_\infty|a^2}{k_0\mu_0\alpha_0},$$

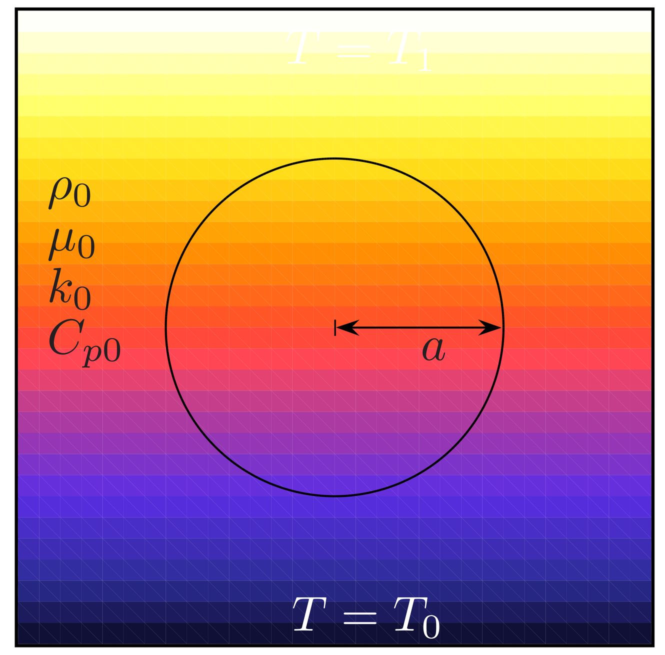

Figure 3: The initial setup of the drop migration problem

We compute the migration velocity and compare with existing literature.

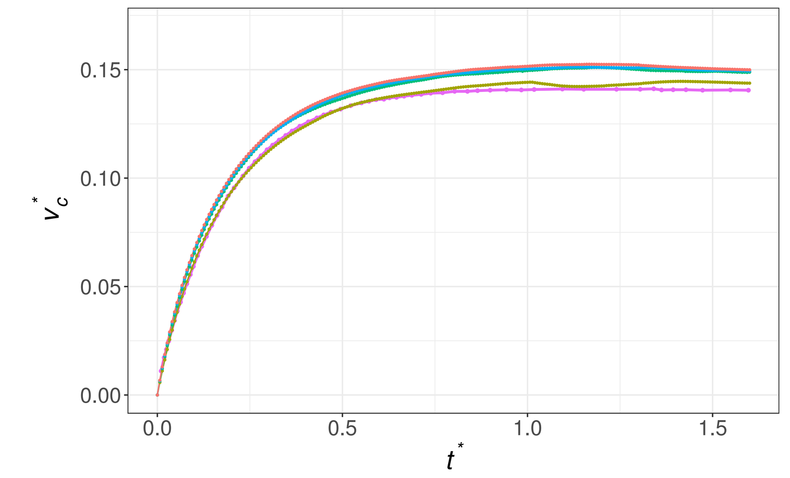

Figure 4: Migration velocity comparison with [?], for $\mathrm{Re} = \mathrm{Ma} = 0.72$, $\mathrm{Ca} = 0.576$

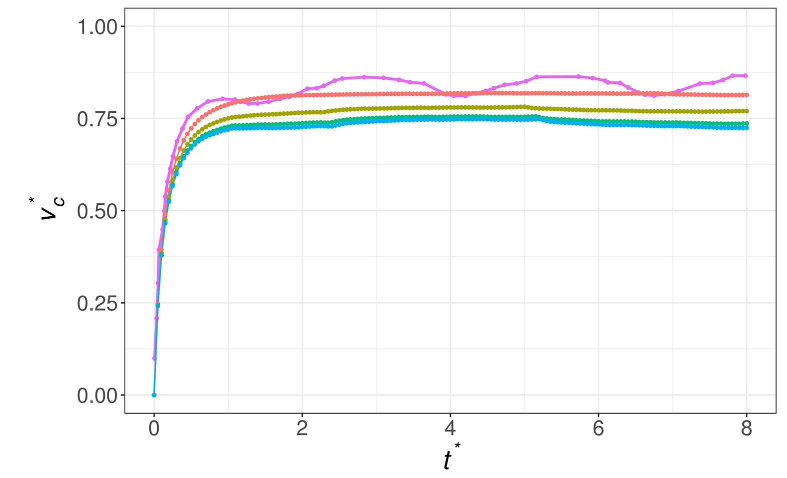

Figure 5: The convergence of the migration velocity as a function of the time step for $\Delta t = 10^{-4}$ (–), 5×10^{-6} (–), 10^{-5} (–), and 5×10^{-6} (–) compared with the results in [?] (–) for 2D simulation, for $\mathrm{Re} = \mathrm{Ca} = 0.066$ and $\mathrm{Ma} = 0$

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